Nonautonomous Dynamical Systems Help Study Long-term Trends and Abrupt Shifts in Climate Variability

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The theory of nonautonomous and random dynamical systems (NDSs and RDSs) provides an appropriate mathematical setting for studying the effects of time-dependent forcing, both natural and anthropogenic, upon a climate system characterized by intrinsic variability [1]. In this theory, the forward attractors of autonomous dynamical systems are replaced by pullback and random attractors (PBAs and RAs) and classical bifurcations by "tipping points." Over the last two decades, these relatively novel concepts have been applied to a number of simple climate models, atmospheric, oceanic and coupled [2].

Important insights into the study of PBAs and RAs arising from climate dynamics have been provided by novel tools from algebraic topology [3,4]. These tools have led to the definition and study of topological tipping points (TTPs), which we present and apply here to simple models of the mid-latitude atmosphere [5,6] and of the wind-driven double-gyre ocean circulation [7].

The atmospheric model is the Lorenz model for seasonal variability [5], while the oceanic one is a low-order approximation of a spectral quasi-geostrophic model for the subtropical and subpolar gyres of the North Atlantic or North Pacific ocean basin, subject to time varying zonal winds [7]. The recent tools from algebraic topology applied to the latter are Branched Manifold Analysis through Homologies (BraMAH) and the Templex, which combines the complex underlying BraMAH with a directed graph that captures the flow in the dynamical system's phase space [4].

The talk will be based on joint work with N. Bodnariuk & G.D. Charó (IFAECI, Buenos Aires, AR), H.A. Dijkstra & B. Maraldi (IMAU, Utrecht, NL), S. Pierini (U. Parthenope, Napoli, IT), D. Sciamarella (IFAECI, Buenos Aires, AR), & S. Speich (ENS & LMD, Paris, FR).

References

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