Internal model techniques of premium and reserve risk for non-life insurers

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Abstract. Solvency II Directive in 2009 has introduced a risk-based solvency requirements for insurance companies across European market. These new requirements will come in force since 1st January 2014 and will be by far more risk-sensitive than Solvency I capital requirements (firstly introduced in the Seventies and only slightly modified in 2002), thus enabling a better coverage of the real risks run by any insurer. Consistent methodologies need to be developed in order to describe both single source of risk and the aggregation between them. Focusing on Non-Life insurers, first results emphasize that technical risk has the greatest impact on the capital requirement. At this regard the main target of this paper is to analyse the risk profile of a multi-line non-life insurer. A risk theoretical simulation model is then applied with the aim to estimate risk capital regarding both Premium and Reserve risk. A comparison has been performed between a Risk Based Capital, obtained by the application of an Internal Risk Model, and the equivalent Solvency Capital Requirement, as provided by the Solvency II standard formula. It is further discussed the dependence problem in order to aggregate losses from different lines of business by different approaches. Numerical results are also figured out in the last part of the paper with evidence of different results for small and medium-large companies coming from Premium risk and Reserve risk pointing out the main reasons of these differences.

Keywords. Solvency II Capital Requirement, Non-Life Underwriting Risk, Premium Risk and Reserve Risk, Aggregation, Internal Model, Solvency II Standard Formula.

M.S.C. classification. 62P05, 91B30, 91B70.
J.E.L. classification. G22, C15.

1 Introduction

As well-known, the Solvency II directive is defining a new framework for a prudential regulation of insurance market in European Union, with particular reference to new capital requirements. At the same time quantitative impact studies (QIS) have been conducted to analyse the impact of the new requirements aiming at calibrating the standard formula for the Solvency Capital Requirement (SCR) and Minimum Capital Requirement (MCR).

As regard to the Standard Formula, a modular approach ([8]) is adopted, which means that individual exposure to each risk category should be assessed in a first step and then aggregated in a second step. Each insurer must decide for itself whether to adopt the standard approach based on market wide or undertaking specific parameter where standardized methods are provided or to use its own (partial or full) internal model subjected to the approval of supervisory authority.

Focusing on the Non-Life Underwriting Risk module only, several studies related to a SCR estimation can be found in the actuarial literature. On one hand, since many years collective risk theory is used to describe the claim amount and to quantify Premium risk for a single line of business and several researches (e.g. [1], [2], [9]) have been developed in order to better describe the risk profile of the single line of business (LoB).

On the other hand, during the last twenty years many stochastic models[1] and closed formulae (e.g. [12], [13]) have been derived with the aim to obtain an estimate of variability for the provision for claims outstanding. Moreover, the actuarial literature started exploring the one-year approach, through closed formulae ([23], [24]) and simulative algorithms based on the so called *re-reserving* approach ([3], [15]), in order to quantify a capital requirement for Reserve risk under a one-year time horizon, consistent with Solvency II.

Finally, several approaches have been tested with the purpose of evaluate the diversification benefit through either linear correlation or copula functions ([4]). At this regard Sandström [19] and Savelli, Clemente [20] dealt with the problem of SCR aggregation methods and proposed alternative methodologies for a skewness adjustment of QIS aggregation formula.

The aim of this work is a comparison between the capital requirements obtained by the standard formula and a partial internal model focusing on only Premium and Reserve risk. CAT and lapse risks are instead neglected. However it is noteworthy that Premium and Reserve have usually a prominent weight on capital charge for non-life underwriting risk.

Focusing only on this sub-module, the last formula, proposed by QIS5, will be analysed highlighting most important assumptions and weaknesses of both *market wide* and *undertaking specific* approaches. Furthermore the capital charge for the same risks will be jointly derived by using an Internal Model too. This inter-

[1] See, at this regard, [6] and [7] for a valuation of the distribution of Chain-Ladder claims reserve through bootstrapping and ODP models. See [14] and [21] for a Collective Risk Model for claims reserve valuation.

nal model follows an approach where both Premium and Reserve risk are dealt with. Model parameters have been calibrated for two different insurers with the aim to allow a full comparison, as much as possible, between SCR obtained by the application of an Internal Model and the capital computed as specified in the QIS5 technical specifications.

It is further discussed the dependence problem in order to aggregate losses from different lines of business and from the two sources of risk by various approaches. At this regard the dependence effect on SCR is examined by comparing the QIS5 aggregation formula with Internal Model results obtained applying either elliptical copula functions or an approximation formula. For these analyses two different non-life insurance companies are also regarded, having different dimensions. The results emphasize that Internal Model gives a reduced risk capital compared to the QIS market-wide standard formula for large insurers whereas small insurers should prefer to use the Standard Formula because, mainly for the lack of a size factor, a reduced required capital is obtained.

2 One-year technical results

We propose here an Internal Model for Premium and Reserve risk for a multi-line Non-Life insurer in order to take into account the characteristics of single risk and line of business and the diversification effect due to the aggregation of either LoBs and risks. In order to introduce the framework, we can define[2] the r.v. $\tilde{Y}_{t+1}$ one-year technical result of the period $(t, t+1)$, evaluated at the end of time t, as the difference between earned premium of the total portfolio with L LoBs and total amount of claims and expenses of the year. For sake of simplicity, we will consider a gross of reinsurance technical result, but we could obviously easily expand[3] previous relation in order to describe the reinsurance effect.
In order to point out main sources of risk, we can directly write:

$$\begin{gathered}\tilde{Y}_{t+1} = \sum_{h=1}^{L} \left(B_{t+1,h}^{writt} + PR_{t,h} - \tilde{PR}_{t+1,h} \right) - \sum_{h=1}^{L} \tilde{E}_{t+1,h} \\ - \sum_{h=1}^{L} \left(\tilde{X}_{t+1,h}^{paid,CY} + \tilde{X}_{t+1,h}^{paid,PY} + \tilde{PCO}_{t+1,h}^{CY} + \tilde{PCO}_{t+1,h}^{PY} - PCO_{t,h} \right). \end{gathered} \tag{1}$$

Earned premiums of a single LoB are here described as the difference between written premium of the year B_{t+1}^{writt} and the one-year change in premium reserve $(\tilde{PR}_{t+1,h} - PR_{t,h})$ for unearned premiums and unexpired risks evaluated under Solvency II criteria. In the same way we takes into account the claim cost of the year, by considering both payments $(\tilde{X}_{t+1})$ for claims and the variation of

[2] From now on, tilde over a letter will indicate a random variable.

[3] Relation can be rewritten by adding a second term that takes into account reinsurance treaties. It considers premium received and claims paid by the reinsurer and other amounts function of the reinsurance form (as commission for quota share treaties). See [9] and [20] for analysis of the effect of excess of loss and quota share treaties on the capital requirement for premium risk.

provisions for outstanding claims ($\tilde{PCO}_{t+1} - PCO_t$) obtained as best estimate (BE) plus risk margin (RM[4]) as defined by Solvency II for non-hedgeable risk.

Moreover, we can split payment for losses between claims incurred during the year $t+1$ ($\tilde{X}^{paid,CY}$) and claims of previous years ($\tilde{X}^{paid,PY}$) and, in the same way, the final provision ($\tilde{PCO}_{t+1}$) can be obtained as the reserve for new claims ($\tilde{PCO}_{t+1}^{CY}$) and the new estimation of reserve for claims already incurred in previous years ($\tilde{PCO}_{t+1}^{PY}$), always summing up best estimate and risk margin. Disregarding, for sake of simplicity, both CAT and lapse risk, we can point out the effect of premium and reserve components:

$$\begin{aligned}\tilde{Y}_{t+1} = & [\textstyle\sum_{h=1}^{L} \left(P_{t+1,h} + \lambda_h P_{t+1,h} + c_h B_{t+1,h} + PR_{t,h} - \tilde{PR}_{t+1,h}\right) \\ & - \textstyle\sum_{h=1}^{L} \tilde{E}_{t+1,h} - \sum_{h=1}^{L} \left(\tilde{X}_{t+1,h}^{paid,CY} + \tilde{BE}_{t+1,h}^{CY} + \tilde{RM}_{t+1,h}^{CY}\right)] \\ & + \left[\textstyle\sum_{h=1}^{L} \left(BE_{t,h} + RM_{t,h} - \tilde{X}_{t+1,h}^{paid,PY} - \tilde{BE}_{t+1,h}^{PY} - \tilde{RM}_{t+1,h}^{PY}\right)\right],\end{aligned} \tag{2}$$

where the first term describes the effect on the technical result of next-year and existing contracts not completely expired. At this regard, gross premiums of single LoB h are defined as the sum of risk premiums equal to the expected amount for claims of current year ($P_{t+1,h} = E[\tilde{X}_{t+1,h}^{paid,CY} + \tilde{BE}_{t+1,h}^{CY}]$), plus safety loading ($\lambda_h P_{t+1,h}$) and expenses loading ($c_h B_{t+1,h}$) equal to the expected amount of expenses ($c_h B_{t+1,h} = E[\tilde{E}_{t+1,h}]$).

The second component, instead, defines the difference between the initial claims reserve and the updated insurer obligations arising during the year $t+1$ for claims incurred before of time t. Insurer obligations are indeed characterized by payments for claims $\tilde{X}_{t+1,h}^{paid,PY}$ and the new estimation of the claims reserve at the end of the year conditionally to the additional information available during the year.

It is now possible to obtain the distribution of the technical result by introducing proper methodologies in order to describe claims and expenses and the aggregation between them. It is easy to show that, under the simplified assumption of written premiums equal to earned premiums, the expected technical result is

$$E[\tilde{Y}_{t+1}] = \sum_{h=1}^{L} \lambda_h P_{t+1,h} + RM_t - E[\tilde{RM}_{t+1}], \tag{3}$$

equal to the safety loading plus the variation between the initial risk margin and the expected value of risk margin at time $t+1$ (considering both the risk margin on the run-off business $\tilde{RM}^{PY}$ and the additional risk margin on the new claims of the year $\tilde{RM}^{CY}$). Only one risk margin is computed within LoBs, by accepting diversification effect as allowed in QIS5.

According to the VaR risk measure at confidence level $\alpha = 99.5\%$ as defined by Solvency II, the following capital requirement (SCR) for Premium and Reserve risk could be derived as:

[4] See [8] for QIS5 definition of Risk Margin and [5] and [18] for more details about risk margin valuation

$$SCR_{\alpha}^{IM} = VaR_{\alpha}\left(\sum_{h=1}^{L}\left(\tilde{X}_{t+1,h}^{paid,CY} + \tilde{BE}_{t+1,h}^{CY} + \tilde{E}_{t+1,h}\right) + \left(\tilde{X}_{t+1,h}^{paid,PY} + \tilde{BE}_{t+1,h}^{PY}\right)\right) - \sum_{h=1}^{L}\left(B_{t+1,h} + BE_{t,h}\right), \tag{4}$$

where we exclude risk margin in SCR as defined in QIS5 (to avoid circularity in Risk Margin evaluation). It is noteworthy that we recognize expected profit and losses in the capital requirement evaluation by considering safety loadings. From our point of view, safety loading should be considered, but it's not clear if it will be allowed in Internal Model validation by the supervisor, because QIS5 standard formula does not regard it in the evaluation.

In the next sections Premium and Reserve risk will be modeled separately and then an aggregate capital requirement will be obtained by introducing a dependence structure.

3 Methodologies for premium risk and reserve risk

Focusing on Premium risk, a Collective Risk Model (CRM) simulation is here applied with the aim to quantify the capital requirement. We denote, for simplicity, with the r.v. $\tilde{X}_{t+1,h}^{CY} = \tilde{X}_{t+1,h}^{paid,CY} + \tilde{BE}_{t+1,h}^{CY}$ the aggregate amount of claims incurred (both paid and reserved) during the year $t+1$ of a single LoB h. Following the collective approach, for each line of business the aggregate claims amount is given by a mixed compound process:

$$\tilde{X}_{t+1,h}^{CY} = \sum_{j=1}^{\tilde{K}_{t+1,h}} \tilde{Z}_{j,t+1,h}, \tag{5}$$

where the number of claims distribution ($\tilde{K}_{t+1,h}$) is the Poisson law, with a parameter $n_{t,h}$ increasing year by year by the real growth rate g_h ($n_{t+1,h} = n_{t,h}(1+g_h)$) and disturbed by a structure random variable $\tilde{q}_h$ distributed as a Gamma with mean equal to 1.
The claim size amounts $\tilde{Z}_{j,t+1,h}$ are assumed i.i.d. with a LogNormal distribution and to be scaled by the claim inflation rate i_h[5].
Different distributional assumptions may be analysed according to claim size but the results under a LogNormal assumption are here only reported.

In order to take into account expenses volatility, we will assume that acquisition and management expenses are described by two random variables with mean and standard deviation respectively equal to $(c_h^A B_{t+1,h}, \sigma_h^A B_{t+1,h})$ and $(c_h^G B_{t+1,h}, \sigma_h^G B_{t+1,h})$. At this regard, a LogNormal distribution has been used in the next case study to simulate expenses too assuming that are not correlated to the claim amount. However the distributional and dependence assumptions have usually not a great impact on the capital charge because of a low volatility

[5] e.g. simple moment of order r is equal to $E[\tilde{Z}_{t+1,h}^{r}] = (1+i_h)^r \cdot E[\tilde{Z}_{t,h}^{r}]$.

of expenses respect to aggregate claim amount. For very specific lines of business (as pecuniary or indemnity and legal insurance), a greater volatility of expense ratio combined to a positive correlation assumption could lead to a noticeable effect on the aggregate capital requirement.

As regard to Reserve risk we derive the capital requirement as the difference between the VaR of insurer obligations at the end of following year less the best estimate of initial reserve. The distribution of the insurer obligation at the end of following year is determined as the sum of the (random) payments occurred in year $t+1$ (i.e. the elements located on the next diagonal of the triangle) and the new claim reserve estimated at time $t+1$ conditionally to additional information available during the new year. As previously explained both claim provisions are allocated without considering risk margin. In the next numerical results, the original version ([7]) of Bootstrapping, based on standardized Pearson residuals and a constant scale parameter, will be applied in order to obtain the estimation variance associated to the claim provision. Process variance has, then, been simulated by testing several parametric distributions whose moments have been calibrated by the incremental values derived by the Bootstrapping procedure. Finally, One-Year claims reserve is obtained by Re-reserving method.

Total capital requirement is determined by using a common Gaussian copulas between different lines of business[6] and between Premium and Reserve risk. Copulas have been calibrated by assuming the same correlation coefficient provided by QIS5[7]. Finally, in the next case study results have been obtained by following the same aggregation order of QIS5 (between risks and then between LoBs) and by assuming a different order based firstly on a aggregation of LoBs and successively between the total capital requirements of each risk.

4 Parameter calibration and main results

This case study provides a comparison between an Internal Model (IM), based on a Collective Risk Model for Premium risk and on a LogNormal Bootstrapping model on a one-year view for Reserve risk, and the QIS5 standard formula (developed in 2010). Both QIS5 market wide (MW) standard formula (based on fixed volatility factors) and QIS5 undertaking (USP) specific formulae have been applied with the aim to derive the capital requirement for Premium and Reserve risk. At this regard approaches 1 and 3 (based on historical pattern of Loss ratios and on CRM) for Premium and the approach 3 (based on Merz-Wuthrich formula) for Reserve have been tested[8].

For these analyses two non-life insurance companies with a different dimension are regarded (their figures are summed up in Table 1). Furthermore both insurers underwrite business in the same 5 lines of business (Accident, Motor

[6] See [20] for a comparison of several aggregation structures on Premium risk and [10] for a diversification analysis on non-life underwriting risk.

[7] QIS5 provides a correlation matrix between LoBs and it assumes a correlation equal to 0.5 between Premium and Reserve risk (see [8]).

[8] See [8] for further details.

Other Damages (MOD), Property, Motor Third-Party Liability (MTPL) and General Third-Party Liability (GTPL)) with the same mix of portfolio (rather similar to the actual proportion in the Italian insurance market).

Table 1. Premium and Reserve Volumes of both insurers

	Omega			Epsilon			Both Insurers		
	$B_{t,h}$	$B_{t+1,h}$	$BE_{t,h}$	$B_{t,h}$	$B_{t+1,h}$	$BE_{t,h}$	$\frac{B_{t,h}}{\sum_{h=1}^{L} B_{t,h}}$	$\frac{BE_{t,h}}{\sum_{h=1}^{L} B_{t,h}}$	$\frac{B_{t+1,h}+BE_{t,h}}{\sum_{h=1}^{L} B_{t+1,h}+BE_{t,h}}$
Accident	100.0	105.0	56.6	10.0	10.5	5.7	10.0%	56.6%	7.0%
MOD	100.0	105.0	19.6	10.0	10.5	2.0	10.0%	19.6%	5.4%
Property	150.0	157.5	109.5	15.0	15.8	10.9	15.0%	73.0%	11.5%
MTPL	550.0	577.5	793.3	55.0	57.8	79.3	55.0%	144.2%	59.2%
GTPL	100.0	105.0	285.7	10.0	10.5	28.6	10.0%	285.7%	16.9%
Total	1,000.0	1,050.0	1,264.7	100.0	105.5	126.5	100.0%	126.5%	100.0%

*Amounts in mln of Euro

Main parameters of CRM have been reported in Table 2. As we can see, both insurers have the same characteristics except for the expected number of claims. OMEGA indeed is ten times larger than EPSILON. It is to be pointed out that some key parameters as safety loading coefficient (λ) and the standard deviation of structure variable ($\sigma_{\tilde{q}}$) are obtained mainly by Italian market Loss Ratios and Combined Ratios for the period 1996-2010[9]. These calibrations will permit a full comparison, as much as possible, between Internal Model and the undertaking-specific approaches. Moreover, expenses parameters have been calibrated by using the historical pattern of both management and acquisition expenses in the same period. It could be noted the lower values of σ^M and σ^A, that will lead to a low variability of expenses producing a low additional capital requirement.
Furthermore, it is to take note that the high figure shown by the safety loading for MOD and Accident, will give rise to a low capital requirement for that single LoB because of high expected technical profits, taken into account in the Internal Model also if expected profits are not allowed in the QIS5 Standard Formula. The claim size CV ($c_{\tilde{Z}}$) is fixed, for each LoB, on the basis of empirical Italian market data. Moreover, the expected number of claims (n_t) and the expected claim cost (m_t) reported in Table 2 for each LoB are referred to the initial year t; they will increase in the examined year $t+1$ (because of a time span of 1 year) as described in the previous section for the dynamic portfolio, according to the annual rate of real growth of portfolio (g) as to number of claims and the annual claim inflation rate (i) as to claim size, assumed to be almost 2% and 3% respectively for all LoBs in the next simulations.

The stochastic model for Reserve risk has been applied to the triangles of incremental paid amounts of each LoB of both insurers. We have not reported

[9] A similar calibration has been computed on a previous paper (see [22]) by using data from 1991 to 2005. Major changes have been observed for the safety loadings and the standard deviation of structure variables.

Table 2. Parameters for Premium Risk

	LOBS	n_t	$\sigma_{\tilde{q}}$	g	m_t	$c_{\tilde{Z}}$	i	λ	c^M	c^A	σ^M	σ^A
Omega	*Accid.*	16,428	15.2%	1.9%	3,200	3	3%	27.7%	4.6%	28.2%	0.3%	0.8%
	MOD	25,900	11.1%	1.9%	2,500	2	3%	13.9%	4.7%	21.5%	0.4%	1.4%
	Prop.	18,849	6.9%	1.9%	6,000	8	3%	-6.4%	4.7%	24.8%	0.6%	0.6%
	MTPL	116,509	8.6%	1.9%	4,000	4	3%	-4.0%	4.7%	14.0%	0.7%	0.8%
	GTPL	8,225	12.8%	1.9%	10,000	12	3%	-13.1%	4.5%	24.0%	0.8%	1.5%
Epsilon	*Accid.*	1,643	15.2%	1.9%	3,200	3	3%	27.7%	4.6%	28.2%	0.3%	0.8%
	MOD	2,590	11.1%	1.9%	2,500	2	3%	13.9%	4.7%	21.5%	0.4%	1.4%
	Prop.	1,885	6.9%	1.9%	6,000	8	3%	-6.4%	4.7%	24.8%	0.6%	0.6%
	MTPL	11,651	8.6%	1.9%	4,000	4	3%	-4.0%	4.7%	14.0%	0.7%	0.8%
	GTPL	823	12.8%	1.9%	10,000	12	3%	-13.1%	4.5%	24.0%	0.8%	1.5%

all the triangles here[10], but a 12x12 triangle has been used. As usual in actuarial literature, the tail, derived by the reserved amount of the oldest accident year at time t, has been added to the payments of the last development year.

Main characteristics of simulated distribution of losses for Premium and Reserve risk and for each loB are reported in Table 3. As regard to Premium risk, CV and skewness of the Aggregate amount of next-year claims plus expenses ($\tilde{X}^{CY} + \tilde{E}$) are summed up. We report, instead, for Reserve risk the characteristics of the distribution of the sum of the payments occurred in year t+1 (i.e. the elements located on the next diagonal of the triangle) and the new claim reserve estimated at time $t + 1$ conditionally to additional information available during the year ($\tilde{X}^{PY} + \tilde{BE}$).

Table 3. CV and skewness of simulated distributions

	Omega				**Epsilon**			
	Premium		*Reserve*		*Premium*		*Reserve*	
	CV	Skew.	CV	Skew.	CV	Skew.	CV	Skew.
Accident	9.49%	0.30	6.30%	0.34	10.52%	0.37	11.26%	0.19
MOD	8.09%	0.21	6.33%	0.26	8.60%	0.22	7.60%	0.41
Property	6.52%	0.92	9.34%	0.57	14.16%	6.52	8.92%	0.45
MTPL	7.18%	0.17	2.84%	0.14	7.76%	0.21	4.94%	0.25
GTPL	13.65%	2.79	8.11%	0.47	31.34%	12.82	15.38%	0.90

Fig. 1 shows SCR ratio obtained by IM as the capital requirement (for Premium or Reserve) divided by initial gross premium volume. As expected for Omega the highest ratios are registered for the line GTPL (65.3% for Premium

[10] Triangles used in the paper can be found on the personal webpage at this link: *http://docenti.unicatt.it/web/scheda_pubblicazione.do?cod_docente=17822&language=ITA&id_pubblicazione=10095&pc_handle=&pc_item_id=0&section=pubblicazioni#.*

and 69.6% for Reserve) due mainly to its large variability (CV=13.7% and 8.1%). Property line shows high ratios too (26.7% and 21.4%), while line MTPL has a ratio for Premium risk (24.8%) significantly greater than Reserve risk (11.1%). The large safety loadings and the low impact of BE on premiums, leads to lower ratios for MOD (11.9% and 3.6%) and Accident (9.1% and 10.6%). Focusing on Epsilon, the effect of pooling risk is clearly noticeable on Premium risk capital charges, while Reserve risk is instead strictly dependent on the characteristics of triangles (for example MTPL and GTPL ratios are almost doubled for the small insurer, while MOD and Property have SCR ratios for Reserve risk similar to Omega).

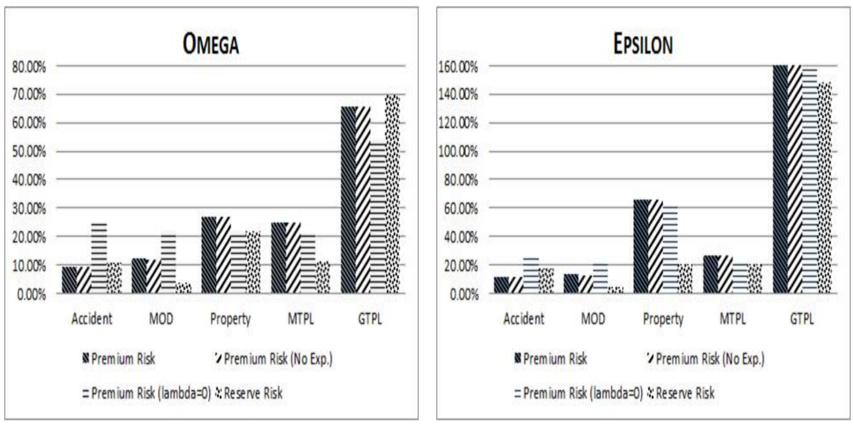

Fig. 1. SCR ratios ($SCR_{t,h}/B_{t,h}$) of Premium or Reserve risk for LoB h

Furthermore, as expected, the effect of expenses is not significant on the capital requirement for Premium risk. Finally, neglecting safety loading (i.e λ equal to 0), SCR is significantly greater for Accident and MOD (where lambda was positive). But at the same way, the choice of QIS5 Standard Formula to not consider safety loading, seems to be less prudential for most important LoBs (see Table 2) but that may be affected by the phase of the underwriting cycle at the evaluation date.

Aggregated distributions reported in Figure 2 for Omega have been derived on the basis of QIS5 correlation matrix. Simulated distributions are indeed obtained by using Gaussian copulas calibrated by these coefficients. Following QIS5 approach the aggregation has been computed firstly between Premium and Reserve risk obtaining the aggregate distribution for each LoB. In a second step, LoBs have been aggregated by deriving the overall distribution whose characteristics are summed up in the last figure. Through the VaR risk measure it has been possible to quantify the capital requirement at the chosen confidence level.

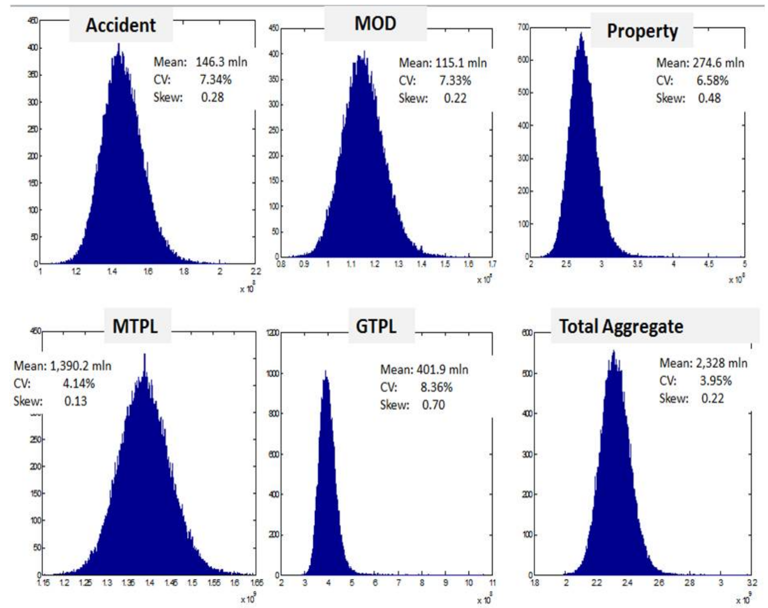

Fig. 2. Omega Insurer - Aggregate Claim Amount distributions of Premium and Reserve Risk for each LoB and for total business (gaussian copula functions)

The aggregate capital requirement is equal to roughly 27% of premiums for Omega and almost 50% for the small insurer (see Fig. 3). Considering a different aggregation order (based firstly on LoBs and then on risks), we derive a slightly lower capital (equal respectively to 26.1% and 47.3%).
Furthermore, it may be observed a greater weight of Premium risk (roughly 60% for Omega and 56% for Epsilon) when an Internal Model is applied. An opposite result is instead obtained when the MW formula is considered. Comparing to QIS5 results, we have that IM leads to lower capital than MW for Omega and greater for Epsilon with a greater saving for Reserve risk for both insurers. Moreover the Internal Model assumes greater values than undertaking specific for both insurers. We have indeed that Internal Model takes into account safety loading, mostly negative, leading to a greater capital than USP. Furthermore the LogNormal assumption of the Standard Formula underestimate the capital requirement for small Insurers. Standard Formula derives, indeed, capital requirement by applying to the volume measure a percentage based on the function $\rho(\sigma)$ as estimate of the distance between the 99.5% quantile and the mean of a Log-Normal distribution with a standard deviation σ.
At this regard, it could be interesting to compare the multiplier derived as the ratio between 99.5% quantile less mean and the standard deviation σ based on the simulated distribution and on the LogNormal assumption. For Omega Company, the LogNormal assumption is not so far from Internal Model results (with a multiplier of 2.69 against 2.74), while in case of Epsilon the LogNormal

assumption underestimates by far the skewness of aggregate claims obtained by simulations (0.18 against an exact skewness of 3.37) and it drives to a multiplier lower than Internal Model (2.77 instead of 3.11).

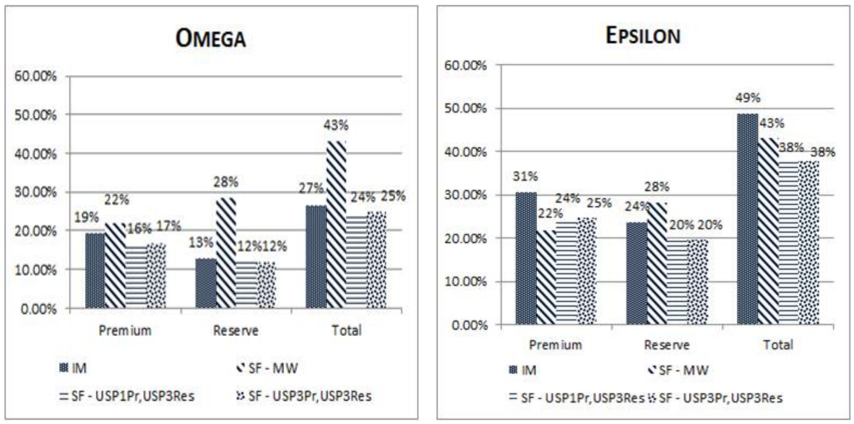

Fig. 3. SCR ratio derived by Internal Model and QIS5 Standard Formula

5 Conclusions

As already mentioned, the main target of this paper is to analyse the risk profile of a multi-line non-life insurer. A risk theoretical simulation model is then applied with the aim to estimate risk capital regarding both Premium and Reserve risk. A comparison has been performed between an Internal Risk Model and QIS5-Solvency II standard formula. Case Studies show a reduction of required capital for large companies with the Internal Model respect to market wide standard formula. The reduction appears larger for Reserve risk. Expected profit (and losses) and stochastic expenses are here considered in Internal Model valuations, while Standard Formula disregards it. Under these assumptions, QIS formula could underestimate the capital requirement. In particular while expenses volatility has a low impact on the needed capital, safety loadings can lead to significant differences.

However, it should be emphasized that Collective Risk Model here adopted is only a simplified version of the complex practical risk management process and furthermore all valuations have been made without considering reinsurance treaties. It is worth reminding that when simulation models are used great care must be paid to avoid as much as possible the three classical modelling risks (model, parameter and process risk). In particular for CRM, the risk of assessing inappropriate parameters, used in the model, assumes a relevant importance for the high impact of some parameters on capital requirement. For example, the standard deviation of structure variable and safety loadings showed the greater changes during a recalibration process (from 1991-2005 to 1996-2010 data).

According to Reserve risk, the choice of the stochastic model and the One-Year approach are representing key approaches to be fully investigated in order to perform evaluations consistent with Solvency II.

Finally, further research improvements will regard the effect of different distributional assumptions on the capital requirement for Premium risk (see [22] for a first analysis). Great care needs to be paid to the choice of a an appropriate claim-size distribution. From our point of view the use of mixture and combined distribution and the valuation of a separate modelling between attritional and large claims are other key issues to be properly analyzed.

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