A model for analyzing the effects of informational asymmetries of the traders

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Abstract. This paper considers a stylized market with heterogeneously informed traders which is a generalisation of the market described in [9]. The aim is to suggest some unusual points of view of more realistic markets that can lead to some interesting comments about the effects of variations of some parameters of the model, in particular the volatility of security's value and the level of transaction costs, on the expected gains of traders and of intermediation system.

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1 Introduction

From the well-known paper by [2], the evaluation of the advantage of more informed traders compared to less informed ones has been widely considered in the literature.

Within this literature a specific role is played by the works in which the fixing price mechanism is endogenous, that is based on the bid and ask prices announced by the traders and remarkable examples of such kind of works are [7], [8], [3], [4], and [5]. One common aim of these works is to underline that in the event of repeated trading the most informed traders, as their profits increase, progressively release their information to less informed traders. In [1] it is analyzed the effect of imitative behaviors by less informed traders about the possibility of reaching equilibria. In [6] it is underlined that the size of bid-ask spread demanded by the traders on a certain risky asset influences the distribution of the returns of such asset.

[9] takes into consideration the market model of a perfectly divisible security in which there is a finite number of traders with different levels of correct information, which means supposedly cost-free. Furthermore, he assumes that each trader is a risk neutral expected gain maximizer and that he is willing to

buy or to sell at most one unit of security at a price that will be fixed by an intermediation system (or intermediary tout court) before the transaction time, while the true value of the security will be common knowledge only after the transaction time.

One of the aims of Schredelseker's paper is to underline that in some cases being "medium informed" brings on worse results than those of more informed ones, and it was only to be expected, but even, and this is really unexpected, than those of less informed ones. The author concludes that for some traders it is profitable not to exploit correct information.

In this paper we consider one of the most significative developments of such a model, which means the introduction of transaction costs which level has to be declared by the intermediation system before the traders fix their bid and ask prices. In Schredelseker's model the intermediary, on the basis of the bid and ask prices announced by each trader (but, let observe that without transaction costs these two prices coincide), fixes the market price with the aim of maximizing the trading volume, while in this paper, in which transaction costs are considered (and in this case, obviously, the bid and ask prices of each trader are not coincident), the intermediary acts on the level of transaction costs with the aim of maximizing the expected value of the collected commissions.

Hence, the aim of this paper is not only to give a contribution about the analysis of the effects of different informational levels in terms of the expected gains of the traders, but also to show how such informational asymmetries act on the profits of the intermediation system.

We remark that whatever the price fixed by the intermediary is, on the basis of the bid and ask prices announced by each trader, the set of traders is divided in three subsets: one of sellers, one of buyers (and not necessarily these two sets have the same cardinality) and the last one of traders that do not participate in trading. As already mentioned, the true value of the security is known only after the transactions time: buyers gain if the intermediary has sufficiently underpriced the security and lose if the intermediary has sufficiently overpriced it. The opposite is true, obviously, for sellers. The word "sufficiently" refers to the fact that the payment of transactions costs by the traders that participate in trading, implies that the algebraic sum of their gains is negative and the opposite figure is just the gain of the intermediary.

With regard to the ability to describe real markets, the model proposed in this paper seems to be a rather fair approximation of the equities opening market. Anyway, the main aim of such a paper is undoubtedly theoretical and it consists in giving an original point of view of economic relationships between more informed traders (paradoxically from insider traders to institutional investors...), less informed traders (from small savers that know all the information given by the media, to less and less informed investors who choose their investments randomly) and the intermediary of the market in which they operate.

The paper is organized as follows. In Section 2 the market model is described: in particular, how the price at which the transaction takes place is defined and how the profits of the traders and of the intermediary are determined. In Section

3 we consider a market with only two groups (in terms of informational level) of traders in order to give some details about the fixing price mechanism and of how the profits of traders and intermediaries are determined. In Section 4 we propose some numerical examples in order to outline the sensitivity analysis of the results respect to variations of the parametrization. In Section 5 some remarks are pointed out.

2 The market model

This section describes the generalisation, with the introduction of transaction costs, of the market model described in [9].

Let α be the level of the transaction costs, (a positive real coefficient, in essence, relatively near to 0) that has to be declared by the intermediation system before traders announce their bid and ask prices. It is obvious that if $\alpha = 0$ the market model without transaction costs is reproduced.

Using p to indicate the market price per unit of security fixed by the intermediary, the transaction costs, which have to be paid by both sellers and buyers, are equal to αp: therefore the intermediary receives $2\alpha p$ for each traded unit.

It is assumed that the true value of the unit of security, V, is unknown before the transaction time. V is given by the number of heads, multiplied by a real positive coefficient c obtained with n independent tosses of a coin that provides the result "head" with probability π. In Schredelseker's model it is $c = 1$ and $\pi = 0.5$.

Hence, the ratio V/c has a binomial distribution with parameters n and π. It holds

$$E[V] = cn\pi \tag{1}$$

and

$$\sigma^2[V] = c^2 n\pi(1-\pi) \; . \tag{2}$$

Let observe that to obtain a generic couple $E[V]$, $\sigma^2[V]$ one of three parameters c, π and n, can be arbitrarily fixed.

About the traders it is assumed that

- the set of traders has cardinality f,
- the aforesaid set is divided at most into $n+1$ groups,
- the generic group h, $h = 0, 1, \ldots, n$, has cardinality f_h, with $f_h \geq 0$ and $f_0 + f_1 + \cdots + f_n = f$,
- each generic trader of the group h, $h = 0, 1, \ldots, n$, knows the result of the first h tosses of the coin (notice that the information held by each trader is "correct"),
- each trader is willing to buy or sell at most one unit of security that is assumed perfectly divisible,
- each trader is a risk neutral expected gain maximizer.

In Schredelseker's market model it is supposed that each trader can use three kinds of strategies: the active, the passive and the contrarian.

The so-called passive strategy provides that the trader not exploit his correct information and, whatever the price may be, he decides to buy or sell by chance, advising the intermediary of his decision before of price fixing. The so-called contrarian strategy provides decisions that are the opposite of those implied in active strategy.

In this paper it is assumed that each trader applies the active strategy. Such strategy provides that if the trader of the generic group h has observed i heads on the first h tosses then he will propose as his bid price

$$b_{hi} = \frac{c\,(i + \pi\,(n-h))}{1+\alpha} \tag{3}$$

and as his ask price

$$a_{hi} = \frac{c\,(i + \pi\,(n-h))}{1-\alpha} \tag{4}$$

since he estimates as expected value of the unit of security

$$v_{hi} = c\,(i + \pi(n-h))$$

and he has to pay αp in transaction costs. With positive α, for each trader the bid price is lower than the ask price, so for $h = 0, 1, \ldots, n$ and $i = 0, 1, \ldots, h$, it is $b_{hi} < a_{hi}$.

Since it is assumed that each trader is risk neutral, if the generic trader of the group h has observed i heads on the first h tosses, $i = 0, 1, \ldots, h$, if p is the market price fixed by the intermediary

- he buys if and only if $p < b_{hi}$
- he sells if and only if $p > a_{hi}$
- he does not participate in trading if $b_{hi} < p < a_{hi}$.

For traders with $p = b_{hi}$ or $p = a_{hi}$ (indifference situations) it is supposed that anyway they prefer to participate in trading and that they respectively buy or sell.

Summarizing, whatever the market price is, no trader is willing to both buy and sell and then the set of traders is divided in three subsets: one of sellers, one of buyers (and not necessarily these two sets have the same cardinality) and the last one of traders that do not participate in trading.

Paradoxically, if all traders have the same informational level and, hence, they announce the same bid and ask prices, then there is no trading: some informational asymmetries are necessary for the existence of the market!

From the previous assumptions it follows that each market scenario is described by a sequence of heads and tails obtainable with n tosses of a coin, and the admissible scenarios are obviously 2^n. Given the level of the transaction costs α and that of the market scenarios, there is a set of intervals of price such that to each of them is associated a constant number of traded units. While in

Schredelseker's model the price is fixed at the middle of the interval for which the number of traded units is maximum, in this paper the price will be fixed in order to maximize the value of the collected commissions and it always takes place in an upper extreme of the aforesaid intervals.

Let observe that the intermediation system fix the market price after the announcement of the bid and ask prices by the traders.

Given a possible scenario, let $\varphi_b(p)$ and $\varphi_s(p)$ be the cardinalities of, respectively, the set of buyers and the set of sellers if the market price is p. It is $\varphi_b(p) + \varphi_s(p) \leq f$ and the number of traded units is just

$$\varphi(p) = \min(\varphi_b(p), \varphi_s(p)) \leq f/2 .$$

If $\varphi(p) = \varphi_b(p)$ then each buyer will buy one unit and each seller will sell $\frac{\varphi_b(p)}{\varphi_s(p)}$ units, while if $\varphi(p) = \varphi_s(p)$ then each seller will sell one unit and each buyer will buy $\frac{\varphi_s(p)}{\varphi_b(p)}$ units. Naturally, in order to maximize his own gain, the intermediary will fix the market price p^* that satisfies

$$p^* = \arg\max_p [2\alpha p \varphi(p)] .$$

After the transactions have taken place at price p, the true value of the unit of security v (v is one of the possible realizations of the random variable V) becomes common knowledge. The profits of the traders that participate in trading and of the intermediary are

- if $v \geq p(1+\alpha)$ (the intermediary has underpriced the security)
  each buyer gains $\dfrac{\varphi(p)}{\varphi_b(p)}(v - p(1+\alpha))$,
  each seller loses $\dfrac{\varphi(p)}{\varphi_s(p)}(v - p(1-\alpha))$,
  the intermediary gains $2\alpha p \varphi(p)$,
- if $v \leq p(1-\alpha)$ (the intermediary has overpriced the security)
  each seller gains $\dfrac{\varphi(p)}{\varphi_s(p)}(p(1-\alpha) - v)$,
  each buyer loses $\dfrac{\varphi(p)}{\varphi_b(p)}(p(1+\alpha) - v)$,
  the intermediary gains $2\alpha p \varphi(p)$,
- if $v \in (p(1-\alpha), p(1+\alpha))$ (the intermediary has fixed a price "near" to the security value)
  each buyer loses $\dfrac{\varphi(p)}{\varphi_s(p)}(v - p(1-\alpha))$,
  each seller loses $\dfrac{\varphi(p)}{\varphi_b(p)}(p(1+\alpha) - v)$,
  the intermediary gains $2\alpha p \varphi(p)$.

We indicate with

- v_r the true value of the unit of security,

- $q_r = \pi^{\frac{v_r}{c}} (1-\pi)^{n-\frac{v_r}{c}}$ the probability to observe v_r,
- p_r the market price,
- φ_r the number of traded units,

relative to the generic r-th market scenario, with $r = 1, 2, \ldots, 2^n$.

Indicating with G_h the random gain of the trader of group h, $h = 0, 1, \ldots, n$, and with g_{hr} the realization of the aforesaid random variable in the r-th market scenario (it is $g_{hr} = 0$ if the traders of group h do not participate in trading), for $r = 1, \ldots, 2^n$, the expected gain of such trader, $E[G_h]$, is given by

$$\gamma_h \triangleq E[G_h] = \sum_{r=1}^{2^n} q_r \, g_{hr} \tag{5}$$

Notice that to obtain the aggregate gain of a group of traders with the same informational level, h, is sufficient to multiply such individual gain, g_{hr}, by the cardinality of the group, f_h.

Indicating with G_I the random gain of the intermediary, and with g_{Ir} the realization of such random variable in the r-th market scenario, for $r = 1, \ldots, 2^n$, it holds

$$g_{Ir} = 2\alpha \, p_r \, \varphi_r$$

from which the intermediary's expected gain is given by

$$\gamma_I \triangleq E[G_I] = \sum_{r=1}^{2^n} q_r \, g_{Ir} = 2\alpha \sum_{r=1}^{2^n} q_r \, p_r \, \varphi_r \,. \tag{6}$$

The aggregate gain of all the groups of traders and of the intermediary has to be null in each scenario, that is, for each $r = 1, 2, \ldots, 2^n$

$$g_{Ir} + \sum_{h=0}^{n} g_{hr} f_h = 0$$

from which it follows for the expected gains

$$\gamma_I + \sum_{h=0}^{n} \gamma_h f_h = 0 \,.$$

In order to measure the distorsive effect generated by the fixing price mechanism previously described, we consider the same index used in [9] that is the variance of the market mispricing. Such index is here given by

$$d = \frac{\sum_{r=1}^{2^n} q_r \, \varphi_r \, (p_r - v_r)^2}{\sum_{r=1}^{2^n} q_r \, \varphi_r} \,. \tag{7}$$

The greater the value of the aforesaid index, the greater the distorsive effect generated by the fixing price mechanism, and we could say the less efficient the market. The denominator of the above ratio is simply the average number of traded units, hereafter indicated with $\overline{\varphi}$.

3 Two groups of traders

For illustrative purposes, but also as an application for special markets, we have taken into consideration a market with only two groups of traders. This example allows us to highlight some details of the fixing price mechanism and of the determination of traders and intermediary's gains.

Let h and k be the two informational levels with $h, k \in \{0, 1, \ldots, n\}$ and $0 \leq h < k \leq n$, so the traders of group k are more informed than the traders of group h. Let f_h and f_k be the cardinalities of such two groups, with $f_h + f_k = f$.

We assume that the traders of group h and those of group k have observed respectively i and j heads, where obviously $i \leq h$ and $i \leq j \leq i + (k - h)$. The bid and ask prices announced by the traders of the two groups are respectively given by according to (3) and (4).

Note that the only two possibilities for the number of traded units are 0 or $\min(f_h, f_k)$. For a transaction to take place when the buyers are the traders of group h and the sellers those of group k then it has to result

$$b_{hi} = \frac{c\,(i + \pi\,(n - h))}{1 + \alpha} \geq \frac{c\,(j + \pi\,(n - k))}{1 - \alpha} = a_{kj}$$

that is

$$(1 - \alpha)\,(i + \pi\,(n - h)) \geq (1 + \alpha)\,(j + \pi\,(n - k)) \tag{8}$$

while, if the buyers are the traders of group k and the sellers those of group h then it has to result

$$b_{kj} = \frac{c\,(j + \pi\,(n - k))}{1 + \alpha} \geq \frac{c\,(i + \pi\,(n - h))}{1 - \alpha} = a_{hi}$$

that is

$$(1 - \alpha)\,(j + \pi\,(n - k)) \geq (1 + \alpha)\,(i + \pi\,(n - h))\ . \tag{9}$$

Notice that for positive α neither (8) nor (9) are satisfied if $j - \pi k = i - \pi h$. Given h and k, for each couple (i, j) with $j - \pi k \neq i - \pi h$ (each market scenario can be univocally associated with couple (i, j), but the opposite is not true), we can determine the highest level of the transaction costs such that the transaction actually takes place. From (8), it has to result for the highest level

$$\alpha \leq \frac{c\,(i - j + (k - h)\,\pi)}{i + j + (2n - h - k)\,\pi}$$

or from (9)

$$\alpha \leq \frac{c\,(j - i + (h - k)\,\pi)}{i + j + (2n - h - k)\,\pi}\ .$$

Let observe that given h and k, the number of the different possible couples (i, j) is $(h + 1)\ (k - h + 1)$ since i can assume $h + 1$ different values and j can assume $k - h + 1$ different values. The probability of the couple (i, j), $P\,((i, j))$, given by

$$P\,((i, j)) = \binom{h}{i} \binom{k - h}{j - i}\, \pi^{i+j}\, (1 - \pi)^{k-(i+j)}$$

is the sum of the probabilities of all the scenarios that imply the couple (i, j) and, hence, the same bid and ask prices.

For illustrative purposes let's consider the case $h = 1$ and $k = 3$, with $f_1 = 3$ and $f_3 = 2$ (and hence $f = 5$). Furthermore, we assume $n = 4$, $\pi = 0.8$, $c = 0.3125$ from which it is $E[V] = 1$ and $\sigma^2[V] = 0.0625$. The bid and ask prices depend on the couple (i, j) of observed heads respectively after the first and after the third toss and they are represented in table 1.

Table 1.

(i,j)	b_{1i}	a_{1i}	b_{3j}	a_{3j}
$(0,0)$	$\frac{0.75}{1+\alpha}$	$\frac{0.75}{1-\alpha}$	$\frac{0.25}{1+\alpha}$	$\frac{0.25}{1-\alpha}$
$(0,1)$	$\frac{0.75}{1+\alpha}$	$\frac{0.75}{1-\alpha}$	$\frac{0.5625}{1+\alpha}$	$\frac{0.5625}{1-\alpha}$
$(0,2)$	$\frac{0.75}{1+\alpha}$	$\frac{0.75}{1-\alpha}$	$\frac{0.875}{1+\alpha}$	$\frac{0.875}{1-\alpha}$
$(1,1)$	$\frac{1.0625}{1+\alpha}$	$\frac{1.0625}{1-\alpha}$	$\frac{0.5625}{1+\alpha}$	$\frac{0.5625}{1-\alpha}$
$(1,2)$	$\frac{1.0625}{1+\alpha}$	$\frac{1.0625}{1-\alpha}$	$\frac{0.875}{1+\alpha}$	$\frac{0.875}{1-\alpha}$
$(1,3)$	$\frac{1.0625}{1+\alpha}$	$\frac{1.0625}{1-\alpha}$	$\frac{1.1875}{1+\alpha}$	$\frac{1.1875}{1-\alpha}$

From these indications it is easy to determine for each couple (i, j) the range of values for α, indicated with $[0, \overline{\alpha}]$, that allows for the transaction between the two groups of traders.

Since in this market the trading is always possible because $j - 3\pi = i - \pi$ is never verified, if the intermediary will fix α in the interval $[0, 0.0556]$ the average number of traded units, $\overline{\varphi}$, reaches its upper limit $2 = \min(f_1, f_3) = \min(3, 2)$.

For illustrative purposes we assume $\alpha = 0.0556$. With the aforesaid choice, the intermediary, in order to maximize his own gain, will decide one case at a time, on the basis of the level of the transaction costs and of the bid and ask prices announced by the traders, the highest price p that allows the transactions. The details are given in the table 3.
We indicate with

- β_1 the profits of each trader of the group 1, for which it is $\beta_1 = \frac{2}{3}(p(1-\alpha) - v)$ if the traders of group 1 sell,

Table 2.

(i,j)	$[0,\overline{\alpha}]$
$(0,0)$	$[0, 0.5000]$
$(0,1)$	$[0, 0.1429]$
$(0,2)$	$[0, 0.0769]$
$(1,1)$	$[0, 0.3077]$
$(1,2)$	$[0, 0.0968]$
$(1,3)$	$[0, 0.0556]$

Table 3.

(i,j)	Buying group	p	$p\,(1+\alpha)$	$p\,(1-\alpha)$
$(0,0)$	1	0.7105	0.7500	0.6710
$(0,1)$	1	0.7105	0.7500	0.6710
$(0,2)$	3	0.8289	0.8750	0.7829
$(1,1)$	1	1.0065	1.0625	0.9506
$(1,2)$	1	1.0065	1.0625	0.9506
$(1,3)$	3	1.1250	1.1875	1.0625

$\beta_1 = \frac{2}{3}\,(v - p\,(1+\alpha))$ if the traders of group 1 buy,
- β_3 the profits of each trader of the group 3, for which it is
  $\beta_3 = (p\,(1-\alpha) - v)$ if the traders of group 3 sell,
  $\beta_3 = (v - p\,(1+\alpha))$ if the traders of group 3 buy,
- β_I the profits of the intermediary, for which it is $\beta_I = 4p\alpha$ in each case.

The details of said gains are underlined in the table 4 in which we report the 3-tuples $\left(i, j, \frac{v}{c}\right)$ that describe both the information known by the two groups of traders and the true value of the unit of security and the associated probabilities $P\left(\left(i, j, \frac{v}{c}\right)\right)$.
In conclusion, with $\alpha = 0.0556$, we have $\overline{\varphi} = 2$ (obviously), $d = 0.0317$ and for the individual expected gains

$$\gamma_1 = -0.1039 \qquad \gamma_3 = 0.0421 \qquad \gamma_I = 0.2274\,.$$

Another choice for the level of the transaction costs could be $\alpha = 0.0769$, that is the highest value that allows the transactions for all couples (i,j), except that for $(i,j) = (1,3)$. With such choice we have $\overline{\varphi} = 0.9760$ (note that it is

Table 4.

$(i,j,v/c)$	$P((i,j,v/c))$	β_1	β_3	β_I
$(0,0,0)$	0.0016	−0.5000	0.6710	0.1579
$(0,0,1)$	0.0064	−0.2916	0.3585	0.1579
$(0,1,1)$	0.0128	−0.2916	0.3585	0.1579
$(0,1,2)$	0.0512	−0.0833	0.0460	0.1579
$(0,2,2)$	0.0256	0.1052	−0.2500	0.1842
$(0,2,3)$	0.1024	−0.1030	0.0624	0.1842
$(1,1,1)$	0.0064	−0.5000	0.6381	0.2237
$(1,1,2)$	0.0256	−0.2916	0.3256	0.2237
$(1,2,2)$	0.0512	−0.2916	0.3256	0.2237
$(1,2,3)$	0.2048	−0.0833	0.0131	0.2237
$(1,3,3)$	0.1024	0.0833	−0.2500	0.2500
$(1,3,4)$	0.4096	−0.1250	0.0625	0.2500

$P((1,3)) = 0.5120)$, $d = 0.040$ and for the individual expected gains

$$\gamma_1 = -0.0640 \qquad \gamma_3 = 0.0285 \qquad \gamma_I = 0.1348\,.$$

Let observe that generally an increase in the level of the transaction costs reduces the average number of traded units and, in this example, it benefits the traders with negative expected gains to the disadvantage of the traders with positive expected gains, and the intermediary.

4 Many groups of traders

The description of a market with many groups of traders with different informational levels requires the use of a considerable number of parameters: n that describes the level of full information, the $(n+1)$-tuple $(f_0, f_1, \ldots, f_n)$ of the cardinalities of the groups of traders with different informational levels (from those who know the results of 0 tosses, that is null information, to those who know the results of n tosses and, hence, that know the true value of the security before the transaction time), c and π that characterize the distribution of the random value of the unit of security, α the level of the transaction costs.

The expected value, $E[V]$, and the variance, $\sigma^2[V]$, of the random value of the unit of security are obtained from n, c and π according to (1) and (2).

In the following examples, for an easier interpretation of the results, we prefer to fix $E[V] = 1$ and to report the value of $\sigma^2[V]$ instead of those of c and π.

In each of the following subsections we propose some numerical examples in order to outline the sensitivity analysis of the results compared to variations of one of the parameters, starting from the reference parametrization

$$n = 6,\ E[V] = 1,\ \sigma^2[V] = 0.1,\ \alpha = 0.005,\ (f_0, f_1, \ldots, f_6) = (7, 6, 5, 4, 3, 2, 1)\ .$$

In each of the following tables we report in the first column the value of the varying parameter, emphasizing in bold type the one relative to the reference parametrization, the average number of the transactions, $\overline{\varphi}$, the variance of the market mispricing, d, defined in (7), the expected gain of the intermediary, γ_I, and the expected gain of the generic trader of the group i, γ_i, with $i = 0, 1, \ldots, n$, overlooking the groups of null cardinality.

4.1 The sharing of the traders in the groups

In order to directly compare the results relative to the average number of the traded units and to the intermediary's expected gain, the total number of traders is 28 in each case.

We remark that the number of traders with a certain informational level could be interpreted as the volume of security exchangeable by an indefinite number of traders with such informational level.

In table 5, fixed $n = 6$, $E[V] = 1$, $\sigma^2[V] = 0.1$, $\alpha = 0.005$, we consider the following nine 7-tuples, F_z, $z = 1, 2, \ldots, 9$, of the cardinalities of the groups of traders $(f_0, f_1, \ldots, f_6)$.

$$F_1 = (7, 6, 5, 4, 3, 2, 1)\ ,\ F_2 = (1, 2, 3, 4, 5, 6, 7)\ ,\ F_3 = (14, 0, 0, 0, 0, 0, 14)\ ,$$

$$F_4 = (0, 14, 0, 0, 0, 14, 0)\ ,\ F_5 = (0, 0, 14, 0, 14, 0, 0)\ ,\ F_6 = (14, 0, 0, 7, 0, 0, 7)\ ,$$

$$F_7 = (16, 0, 0, 8, 0, 0, 4)\ ,\ F_8 = (4, 0, 0, 8, 0, 0, 16)\ ,\ F_9 = (4, 4, 4, 4, 4, 4, 4)\ .$$

Note that with the sharing F_1 the reference parametrization is reproduced.

A significative remark is that, as in [9], the expected gain of the traders with medium-low informational levels (e.g. those of groups 1, 2 and 3) are, in some cases, worse than those obtained by less informed ones (even of those obtained by traders of groups 0 that have null information). On the contrary, the expected gains of the most informed traders (e.g. those of groups 5 and 6) are always the highest and they are positively stable.

About the expected gain of the intermediary, let observe how the most profitable sharings seem to be those in which the traders are divided into two groups (in particular when the gap of informational levels between the two groups is large): in the aforesaid case, in each market scenario, one of the two will be the group of sellers and the other the group of buyers and the average number of traded units reaches the maximum admissible value, that is $f/2 = 14$.

Notice that the lower is d, which means the more efficient is the market, the higher are the gains of the less informed traders and the lower are the gains of the most informed ones.

Table 5.

F_z	$\overline{\varphi}$	d	γ_I	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
$\mathbf{F_1}$	12.676	.060	.129	-.090	-.101	.027	.052	.104	.142	.169
F_2	12.644	.016	.130	-.035	-.047	-.061	-.062	-.059	.023	.084
F_3	14.000	.052	.157	-.136						.124
F_4	14.000	.051	.154		-.116				.105	
F_5	14.000	.050	.150			-.083		.072		
F_6	12.486	.073	.134	-.143			.104			.164
F_7	11.133	.073	.119	-.099			.099			.168
F_8	11.133	.022	.120	-.065			-.087			.052
F_9	12.000	.036	.126	-.071	-.084	-.072	-.035	.037	.077	.116

4.2 The level of full information

Fixed the 7-tuple $(f_0, f_1, \ldots, f_6)$, we consider increasing values of the level of full information (which means increasing values of n) without adding any traders: the relative information level of each group decreases and the aforesaid decrease is relatively greater for the groups of the most informed traders compared to the groups of the less informed traders.

In table 6, fixed $E[V] = 1$, $\sigma^2[V] = 0.1$, $\alpha = 0.005$, $(f_0, f_1, \ldots, f_6) = (7, 6, 5, 4, 3, 2, 1)$, we consider some values of n, that are reported in the first column, assuming $f_h = 0$ for each $h = 7, 8, \ldots n$.

Table 6.

n	$\overline{\varphi}$	d	γ_I	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
6	12.676	.060	.129	-.090	-.101	.027	.052	.104	.142	.169
9	12.642	.074	.129	-.070	-.079	.009	.044	.081	.114	.134
12	12.651	.080	.128	-.067	-.063	.006	.045	.070	.090	.114
15	12.506	.085	.127	-.062	-.061	.010	.040	.065	.083	.103

As expected, the obtained results point out that the reduction of all relative information levels (that is when n increases) benefits the traders with low informational levels (those with negative expected gain) to the disadvantage of, especially, the most informed ones (those of groups 4, 5 and 6) while the expected gain of the intermediary seems rather stable.

It is quite clear that a reduction of all relative information levels makes the market less efficient, at least in terms of the index d, while the average number of traded units is hardly affected by variations of said parameter.

4.3 The variance of the random value of the security

In this subsection it is proposed the analysis of the impact of volatility which is commonly regarded as the key parameter of the financial markets and which, in this paper, is expressed by the variance of the random value of the security.

In table 7, fixed $n = 6$, $E[V] = 1$, $\alpha = 0.005$, $(f_0, f_1, \ldots, f_6) = (7, 6, 5, 4, 3, 2, 1)$, we consider some values of $\sigma^2[V]$ that are reported in the first column.

Table 7.

$\sigma^2[V]$	$\overline{\varphi}$	d	γ_I	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
.001	0.236	.002	.002	-.002	-.001	.000	.001	.002	.003	.004
.010	10.234	.007	.103	-.016	-.033	-.014	.015	.029	.042	.052
.050	12.840	.032	.130	-.065	-.086	.021	.049	.076	.095	.117
.100	12.676	.060	.129	-.090	-.101	.027	.052	.104	.142	.169
.200	12.599	.123	.131	-.131	-.127	.021	.097	.149	.187	.237
.300	12.621	.183	.134	-.161	-.173	.048	.118	.186	.233	.292

The most interesting remark regards the fact that the higher the variance, the higher the expected gains of the more informed traders (ones of the groups 2, 3, ..., 6) and of the intermediary and the lower the expected gains of the less informed traders (ones of the groups 0 and 1). This remark suggests that there could be a common interest of the more informed traders and of the intermediary in making the volatility as high as possible, to the disadvantage of the less informed traders.

Furthermore, if the variance tends to 0 (let observe for example the case in which it is $\sigma^2[V] = 0.001$), that is if the value of the security tends to be deterministic, then the trading volumes tend to be null, with obvious consequences on the profits of the traders and of the intermediary. Paradoxically, a null variance implies the elimination of all informational asymmetries and, hence, of the conditions of trading: the presence of some uncertainty is necessary for the existence of the market!

Notice that the higher the variance, the higher the difference between the bid and ask prices of each trader and hence, it seems quite reasonable to say that an increase of the variance tends to increase the mispricing between the market price and the true value of the security and hence, to reduce the efficiency of the market.

4.4 The level of the transaction costs

In this subsection we analyze the effects relative to variations of the level of transaction costs, which is the parameter managed by the intermediary in order to maximize its expected gain. Such expected gain depends on the trade-off between the gain relative to a single traded unit and the average number of traded units when the level of the transaction costs increases.

In the table 8, fixed $n = 6$, $E[V] = 1$, $\sigma^2[V] = 0.1$, $(f_0, f_1, \ldots, f_6) = (7, 6, 5, 4, 3, 2, 1)$ we consider some values of α that are reported in the first column.

Table 8.

α	$\overline{\varphi}$	d	γ_I	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
.0010	12.676	.060	.026	-.085	-.097	.030	.055	.106	.145	.172
.**0050**	12.676	.060	.129	-.090	-.101	.027	.052	.104	.142	.169
.0100	12.676	.060	.258	-.095	-.107	.023	.048	.100	.137	.164
.0900	7.085	.062	1.267	-.139	-.074	-.053	.011	.045	.073	.096
.0909	7.085	.062	1.279	-.140	-.074	-.053	.010	.044	.072	.095
.0910	6.277	.066	1.133	-.125	-.073	-.040	.003	.045	.071	.096
.0967	6.277	.066	1.198	-.128	-.074	-.043	.000	.042	.069	.093
.0968	6.051	.066	1.156	-.131	-.080	-.020	.000	.042	.064	.091

As expected we can observe that the intermediary's interests are the opposite of those of the traders, when an increase of the level of transaction costs does not imply a decrease of the expected number of traded units. On the contrary, when an increase of this parameter implies a decrease of the average number of traded units, the expected gains of the most informed traders (e.g. ones of groups 4, 5 and 6) decrease quite univocally, while some traders with negative expected gains reduce their losses.

In this example the trade-off between the gain relative to the single traded unit and the average number of traded units is profitable for the intermediary until the level of the transaction costs reaches the value 0.0909, point in which intermediary's expected gain is maximum. It has to be underlined that the aims of maximizing the average number of traded units and of maximizing the intermediary's expected gain do not agree.

In order to emphasize the importance of managing the level of transactions costs for maximizing the expected gain of the intermediation system, we report the level, here indicated by $\alpha_{\max}$, which maximizes the expected gain of the intermediation system, $\gamma_{I_{\max}}$, for some sharings of the traders in the groups considered in section 4.1.

Table 9.

F_z	$\alpha_{\max}$	$\gamma_{I_{\max}}$
$\mathbf{F}_1$	0.0909	1.279
F_2	0.0943	1.226
F_3	0.1428	1.847
F_7	0.1272	1.710
F_8	0.1267	1.446
F_9	0.0877	1.334

Let observe that the level of transaction costs which maximizes the expected gain of the intermediation system takes a wide range of values. This result suggests that the intermediation system should pay attention in choosing profitable levels for each kind of market.

As it was expected, the intermediation system obtains the highest expected gain when the traders are divided in two groups with the maximum difference in terms of informational levels (sharing F_3): such two groups tend to have big differences in their expectations of security's value, and so in many cases trading is attainable also with high levels of transaction costs.

Obviously, the analysis on the level of transaction costs which maximizes the expected gain of the intermediation system could be interesting also for variations of each other parameter of the model.

5 Conclusion

Although we have considered a rather stylized market model, this paper can suggest some unusual points of view of more realistic market models in which heterogeneously informed traders operate, which could lead to some interesting remarks on financial markets analysis.

Indeed, as in [9], the result is that the expected gain of the traders with intermediate informational levels are, in some cases, worse than those obtained by less informed ones. For example, such intermediate levels could be those of small savers who try to gather as much information as possible from the media which, unexpectedly, could not lead to better results than those obtained by less meticulous small savers. On the other hand, as already mentioned, some informational asymmetries are necessary for the existence of the market.

We have also underlined that the intermediation system should pay attention on managing the level of transaction costs, which has to declared to the traders before they fix their ask and bid prices, in order to maximize its expected gain, as suggested by the results proposed in section 4.4.

Another remarkable suggestion of this paper concerns the impact of volatility. The results obtained show both that when volatility is too low, it reduces the trading volume dramatically, and that the more informed traders and the intermediary could have a common interest in increasing volatility, since this increases their expected gains.

However, they should consider that from the point of view of the interpretation of the model in terms of the Capital Asset Pricing Model theory, the negative expected gains obtained by less informed traders have to be interpreted as partial reductions of extra gains attainable in this "risky" market respect to those attainable with the free-risk return. Indeed, it suggests that they should "control" that volatility not exceed the maximum level such that the traders with the worst expected gain prefer to operate in this "risky" market rather than investing in the free-risk return: on the other hand, the aggregate positive gain of the most informed traders and of the intermediary would be, consequently, smaller.

Considering that the most informed traders could be identified as the institutional investors (which, as it is commonly accepted, manage the greater share of the trading volume), it is not difficult to imagine that volatility could be somehow "controlled" by the aforesaid traders in the light of previous remarks.

Some other developments of the model described in [9] that could be interesting to analyze are the consideration of risk adverse traders and the introduction of the cost of the correct information.

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