Mergers, acquisitions, and innovation*

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Abstract. Evidence shows that the merger firms are more successful in R&D than those that are not. The question that then arises is how many firms should the merger firms invest in while attempting to acquire innovation. We derive a unique and closed-formed firm-level profit maximizing number of start-up entrepreneurial firms that the merger firms take equity positions in while attempting to acquire innovation. The model is mainly described by two stochastic differential equations. For each stochastic differential equation, we apply the Bayesian inferences to the construction of expectation on R&D process and to the rise of profit excluding acquisition-related costs resulting from marketing.

Keywords. Stochastic differential equations, Bayes' rule, innovation.

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J.E.L. classification. C15, L20, M21.

> [T]here is little justification for monopoly in a world of Open Innovation. – [10, p.194]

1 Introduction

Open Innovation is essentially a paradigm that generates ideas by several firms or acquires innovation through acquisition or capital investment. Chesbrough ([10]) insists that innovation performed by only one firm is inefficient. Evidence by Griliches ([18]) estimates R&D and productivity at the firm level and it reveals that R&D investments of the merger firms are more successful than those of non-merger firms. Griliches [18, chap.5, pp.113-117] states that

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> About one firm out of five in our "complete" sample... appeared to be affected... by considerable and generally simultaneous "jumps"... in gross plant, number of employees, and sales. We have been able to check and convince ourselves that most of these jumps do, in fact, result from mergers, although some may be the result of very rapid growth.... One way of dealing with this problem is simply to drop the offending firms. This results in what we have called the "restricted" sample.... The within estimates are... very sensitive, and the estimated γ collapses, declining from .11 to .05 and -.03 in the complete, intermediate, and restricted samples, respectively [where γ is the elasticity of output with respect to R&D capital].... It is clear... that the merger firms are responsible for the difference.... In other words, R&D seems most effective for firms growing rapidly through mergers, and both phenomena (mergers and R&D growth) are apparently related.... Such a finding raises questions that deserve additional analysis: Who are these "merger" firms and why would their R&D investment be more successful? What kind of selectivity is at work here? [The phrase within square brackets is added by the author]

Our motivation of this paper is to derive answers to the questions posed by Griliches. The purpose of this paper is to solve an optimal number of start-up entrepreneurial firms that the merger firms take equity positions in so as to maximize its profits while attempting to acquire innovation within a specified period of time. In this paper the merger firm is a firm that is planning to merger a start-up entrepreneurial firm or the firm itself after the merger. An equity position is an equity investment made by the merger firms for the purpose of acquiring 50 percent or more of the shares issued by the start-up entrepreneurial firms after their IPO, the success of their R&D projects, or innovation. We assume that the merger firms employ equity positions to finance them obtained by the payment that equals the value of the call-option price multiplied by more than 50 percent of the shares issued per start-up entrepreneurial firm if the start-up entrepreneurial firm has been published.

In our model, we consider the cases where only the merger firms and the start-up entrepreneurial firms are involved. Since the start-up entrepreneurial firms are relatively small (before the IPO), the shareholders of the start-up entrepreneurial firms are only the merger firms or the owners of the start-up entrepreneurial firms. Hence, shareholder disapprovals do not occur once the merger announcement (contract) has been made for there is no one who opposes to the merger deal. And because the start-up entrepreneurial firms are relatively much smaller than the merger firms, regulatory considerations such as anti-monopoly are not required.

Models that analyze acquisition and innovation have begun by Aghion and Tirole ([5, 6]) using the framework of Grossman and Hart ([19]). It is second-best to purchase the other firm when the firm's investment is relatively larger than the other's [19]. Aghion and Tirole ([5, 6]) study the integrated case and nonintegrated cases. The former is the case the customer owns the research unit. Herein, the customer owns and freely uses the innovation developed by the research unit. We analyze the integrated case when there are several start-up entrepreneurial firms (research units) to finance prior to the success of R&D. The merger firms (the customers) freely use the innovation developed by the research unit after the former own the latter. Aghion and Howitt

([3]) provide more complex considerations to the study of Aghion and Tirole ([5, 6]) by introducing rent-sharing between researchers and developers. By incorporating the concept of researchers and developers (in their model, researchers after becoming start-up entrepreneurial firms, hire developers for applied research that is necessary for the product to be sold), they study the positive effect of competition on growth. Klepper ([24]) considers how entry, exit, market structure, and innovation - the ratio of product to process innovation - vary from the birth of industries through maturity. Arguing that firm innovation and firm growth, entry, exit, and size distribution deserve an integrated treatment, Klette and Kortum ([25]) capture and analyze in their model heterogeneous firms, simultaneous exit and entry, optimal investments in expansion, explicit individual firm dynamics, and a steady-state firm size distribution. Loury ([30]) investigates the relationship between the market structure and innovation through a model of non-cooperative game. He analyzes that the shape of relationship between the aggregate success rate of rivals and firm's optimal investment in R&D is inverted U. Lee and Wilde ([27]), using a model similar to that of Loury ([30]) which assumes the reward to be the first to introduce the new technology is a fixed sum, where in Loury ([30]) that is a flow, study the positive relationship between the aggregate success rate of rivals and the firm's optimal investment in R&D.

Several papers, including Cowan ([12]), insist that the relationship between the number of firms and the total industry R&D is an inverted-U shape. Using the model that is a discrete version of the model proposed by Loury ([30]), Cowan ([12]) suggests that an increase in the number of firms in an industry decreases the number of R&D projects undertaken by per firm. This causes a decline in the knowledge that is generated by those R&D projects. On the basis of this fact, he shows that the relationship between the number of firms in an industry and the rate of technological development - the total number of R&D projects undertaken in an industry - is an inverted-U shape. Aghion et al. ([1]) argue that the relationship between product market competition (PMC) and innovation is an inverted-U shape, implying that there exists an optimal competition for the greatest innovation. In this model, they use "escaping competition" as an incentive for engaging in innovation.

Both Cowan ([12]) and Aghion et al. ([1]) assume that the total number of R&D projects undertaken in an industry M is not given and that it is a function of the total number of firms in an industry. In this paper, we assume that M is constant in the short term regardless of whether or not there is a change in the number of firms. This is because there is an upper bound to the number of researchers that firms contract with. Firms that attempt to innovate, contract only with energetic researchers who are growing. These researchers face time constraints. The time constraints of the researchers and the upper bound of the number of researchers restrict M. We assume that these requirements of firms for researchers do not change in the short term. Therefore, we assume that M is constant in the short term. Later, we refer to this in detail and in Section 2, we refer to the longer term when there is a possibility of variation in M.

Section 2 presents our model that is mainly comprised by two stochastic differential equations. The optimal number of start-up entrepreneurial firms that the merger firms take equity positions in for profit maximization is also derived in this section. Section 3 simulates the derived optimal number of firms that the merger firms take equity positions in. Section 4 presents a brief review of the results and the conclusions that discuss the selectivity of the merger firms.

2 The model

2.1 Assumptions

For companies like Intel and Hewlett-Packard, it makes perfect sense to invest substantially in options, such as equity investments in small entrepreneurial companies with interesting technology [32].

In this paper, the merger firms take equity positions in n start-up entrepreneurial firms while attempting to acquire innovation. The equity investment c for the each start-up entrepreneurial firm i has the value of the call-option price multiplied by more than 50 percent of the shares issued per start-up entrepreneurial firm if the start-up entrepreneurial firm is published. Thus, after the publication of the start-up entrepreneurial firms, the merger firms enjoy equal footings with regard to the right to purchase the start-up entrepreneurial firms based on the "exercise price" of call options. Profits excluding acquisition-related costs $x(t)$ of the merger firms and the number of successful R&D projects $j(t)$ in time t are given by stochastic differential equations, where acquisition-related costs are $nc + C$ and C has the value of the call-option strike price multiplied by more than 50 percent of the shares issued per start-up entrepreneurial firm if the start-up entrepreneurial firm is published. The merger firms acquire only one start-up entrepreneurial firm from n that has the best innovation capabilities. As stated in Loury ([30]), there are no externalities in the R&D process (for example, no stealing of trade secrets). As noted above, M is constant in the short term. Firms are risk neutral. Demand varies stochastically. The merger firms are in the monopolistically competitive product markets under complete information and seek for their short term profits as myopic firms. They can raise as much funds as they require from the financial sector at the constant short-term interest rate r. In the financial sector, complete stock markets with the Black-Scholes ([8]) economy also prevail. Thus, there are no budget constraints. There are no gains or losses from the acquisition itself, owing to the no-arbitrage condition. Therefore, we can focus our study on the profit results from innovation and not from the arbitrage of the acquisition. The gained assets, including intangible and tangible assets (net assets), are arbitrage free owing to the complete stock markets. Hence, acquisition-related costs have no effects on the stock prices $S(t)$ of the merger firms.

If the merger firms have the right to purchase the stocks of start-up entrepreneurial firms that have succeeded in their R&D projects prior to the success with the call options, the risk of investments can be minimized by setting the "exercise price" beforehand.

2.2 Number of times of successful R&D projects

In this section, we present a drift derived from Bayes's Rule founded on the Poisson distributions in order to express the increase in $j(t)$. The success rate of R&D projects depends on the increase in $j(t)$ by his/her past experiences in them. The researcher's past experiences of success in R&D positively affects $j(t)$. Hence, they increase the success rate of R&D projects. We use the Poisson arrival rate of an innovation. Using Bayes' Rule founded on the Poisson distributions we get

$$\lambda'' = \frac{j''}{m''} = \frac{j' + j}{m' + m} \tag{1}$$

[38, pp.345-362],[1] where λ'' is the posterior distribution, λ is the Poisson arrival rate of an innovation, and j' and m' are the prior distributions. Further, $m = \frac{M}{N} = \frac{M}{n+k}$, where m is the number of times of R&D projects that are evenly allocated for each i and t by the merger firms, $N = n+k$ is the total number of firms in an industry, and k is the number of other firms that the merger firms did not take equity positions in including the number of rivals of the merger firms. In other words, k is the sum of the number of the start-up entrepreneurial firms that the merger firms have no technological interest in and that of firms that are rivals to the merger firms. Using the update of Bayes's Rule founded on the normal distributions ([9, p.25]), the update of j and m by the prior distributions will be

$$\begin{aligned} j''' &= j'' + j \\ &= 2j + j', \end{aligned}$$

where j''' is the posterior distribution after 2 updates. Assuming that $j' = j$, and applying the same to m,

$$\begin{aligned} \lambda^{(t)} &= \frac{tj}{tm} \\ &= \frac{j}{m}. \end{aligned}$$

Since, m is evenly allocated for each t, the increase in the success rate of R&D projects depends on j. That is, if $j(t)$ grows, the expression of the positive effects of previous successful R&D experiences on the number of times a researcher succeeds will be[2]

$$dj(t) = \mu j(t)dt + j(t)\sqrt{\frac{j(t)}{n}}dV(t), \quad \mu \geq g\ . \tag{2}$$

In (2), μ is the growth rate of $j(t)$, the know-how, the experience required to succeed in R&D gained by the researchers and $V(t)$ depicts the Brownian motion of $j(t)$. We

[1] Pratt, Raiffa, and Schlaifer ([38]) derive (1) by assuming that the prior distribution is the gamma-1 distribution. The definition of the function is given in Pratt, Raiffa, and Schlaifer ([38, p.202]). See Loredo ([28]) and Rainwater and Wu ([40]) for the theoretical frameworks that derive exactly the same equation as the gamma-1 distribution. Gregory ([17, pp.376-378]) derives an application of it based on Loredo ([29]) with some theoretical expansion. Both Loredo ([28, 29]) and Gregory ([17]) employ the Jeffrey's prior as the prior distribution for deriving it. See Jaynes ([21]) for a general discussion on the Jeffrey's prior.

[2] See the Appendix for proofs of the existence and uniqueness of a solution of (2).

employ the Brownian motion since the researchers' efforts are the ones that are continuous, engaging in m R&D projects in each t. Though the successful R&D seems to be discontinuous process, for the researchers themselves, it is a thing that can only be accomplished by conducting numerous R&D projects including the ones that fail. Numerous R&D projects have the role as a path to the success. Thus, we assume that $j(t)$ is a continuous process and apply Brownian motion to it. And $j(t)$ does not jump. To see why, we use contradiction. Suppose that $j(t)$ jumps. Then, from $(A10)$ in the Appendix, the stock price of the merger firm $S(t)$ also jumps. Because "such a process allows for a positive probability of stock price change of extraordinary magnitude ([34, pp.126-127])," there would be arbitrage. And thus, this is not Pareto optimal, contradicting the conclusion of the first fundamental theorem of welfare economics or first welfare theorem.[3] Therefore $j(t)$, especially those that are in n, do not jump. This is because the risk induced by the jumps cannot be hedged and correspondingly, the economic model is not complete ([7, p.1834]), which contradicts with our assumption of complete stock market.

Since, firms have requirements for the researchers to undertake R&D, they only contract with the researchers with $\mu \geq g$, where a constant g in the short term is the lower bound of μ. Firms only contract with the researchers who are within their bounds that reflect their qualification criteria for researchers. However, when a depression occurs in the longer term, their qualification criteria for researchers become severe and their bounds become narrower, that is, g becomes larger. Thus, during depressions in the longer term, M decreases. Therefore, M is a negative function of g in the long term. Notice that in the short term, M is constant as $\bar{M}$. The reasons for $M = \bar{M}$ are as follows. One of the reason is that firms do not change their standards to ensure consistency in contracts. Thus, firms' g do not vary in the short term. The difference between the short term and the long term, in this paper, is whether it is shorter or longer than a specified period of time $[0, T]$. That is, the term $[0, t]$ is

$$[0, t] = \begin{cases} \text{short term}, & t \leq T \\ \text{long term}, & t > T \end{cases},$$

where T is the maturity date. Since researchers with $\mu < g$ are not considered for engaging in R&D undertakings, there are upper bounds to the numbers of researchers that a firm contracts with in the short term. And these researchers that suffice firms' requirements face time constraints. The number of researchers that firms contract with and the time constraints faced by researchers restrict M. We also assume that the economy employs all these resources, that is, the researchers' labor markets clear. Thus, $M = \bar{M}$ in the short term. And μ does not depend on n. The start-up entrepreneurial firms that the merger firms took equity positions in do not share their researchers' know-how μ with each other because they are rivals in terms of receiving the reward (C) from the merger firms after the success of R&D. Each start-up entrepreneurial firm that receives equity finance from the merger firms do not have any incentive to share the information about its μ with each other. Hence, μ is not a function of n.

Since the discoveries of the researchers are actual facts, they vary stochastically.

[3] See Mas-Colell, Whinston, and Green ([33, p.694]).

This is expressed by $\sqrt{\frac{j(t)}{n}}dV(t)$, where we are use the variance of $j(t)$ is $j(t)$ considering the variance of the Poisson distribution $\lambda = \frac{j(t)}{m}$ and m allocated evenly for each t. $\sqrt{\frac{j(t)}{n}}$ is the volatility.

In the next section, we present a drift of $x(t)$ derived from the inverse of an equation derived from Bayes's Rule founded on the normal distributions.

2.3 Marketing

We assume that firms perform marketing activities for the new product depending on the invention. In this paper, these marketing activities contribute to the largest $x(t)$ through the *four Ps*, i.e. Product, Price, Place, and Production [26, pp.32-33]. In our model, demand varies stochastically. The price that maximizes sales depends on the price elasticity of demand. Assume that the increment of the inverse of the equation derived from Bayes' Rule founded on the normal distributions per time to be the drift of $x(t)$. This is expressing the augmentation of precision per time with respect to the true values of the four Ps that generates the largest $x(t)$ through marketing activities. Then, we get the incremental process of $x(t)$ as[4]

$$dx(t) = \frac{\tilde{m}n}{\sigma^2}x(t)dt + x(t)\frac{\sigma}{\sqrt{n}}d\tilde{V}(t) \ , \tag{3}$$

where $\tilde{m}$ is the number of times that marketing activities that are evenly allocated for each i and t by the merger firms and $\tilde{V}(t)$, the Brownian motion of $x(t)$. The second term in the RHS of (3) is derived by assuming that the merger firms disperse variance by investing equally in each i. Because, $x(t)$ is an actual event that is subject to fluctuation this is expressed by $\frac{\sigma}{\sqrt{n}}d\tilde{V}(t)$. $\frac{\sigma}{\sqrt{n}}$ is the volatility. Further, $V(t)$ and $\tilde{V}(t)$ are independent of each other, since the former Brownian motion is the fluctuation of $j(t)$ and the latter, the fluctuation of $x(t)$. (3) is expressing the increment of $x(t)$ whose growth rate is depicted by the augmentation of precision per time with respect to the true value of the four Ps that generate the largest $x(t)$.

In this paper, $\tilde{m}$ and m are assumed to be proportional to each other. This is because one of the functions of marketing is to determine the directions of technologies before it is too late to change the characteristics of them. The more R&D projects are conducted, the more difficult it is to change the directions of new technologies. Several authors emphasize the importance of communication between the marketing and R&D departments. "[C]ompanies serious about competing via innovation must recognize that the educational task is multidirectional; urging R&D to educate marketing but not vice versa is a mistake[20, p.220]." Further, "[w]ithout close communication between marketing and R&D, the successful new brand development rate will be even lower than the pitiful national average of 10 to 30 percent[31, p.81]." Thus, we assume that increases in m increase $\tilde{m}$. Therefore m and $\tilde{m}$ are conducted at the same pace. While deriving the optimization, we assume that $m = \tilde{m}$ due to the above mentioned reasons and the simplicity of calculations. And here lies the "selectivity" of the merger firms that we

[4] See the Appendix. Also see the Appendix for proofs of the existence and uniqueness of a solution of (3).

will discuss more in Section 4. Moreover, $m \not< \tilde{m}$ since we adopt Poisson distribution. We need m to be large enough to make $\frac{j}{m}$ small. We do not discuss the allocation of resources between the marketing and R&D departments since this problem deviates from our thesis.

2.4 Expected profits of the merger firms

From (1), the expected profits of the merger firms π becomes[5]

$$\pi = n\frac{\bar{j}(t)}{m}x(t) - nc - C\ , \tag{4}$$

where $\bar{j}(t) = \frac{1}{n}\sum_{i=1}^{n} j_i$. We assume that the start-up entrepreneurial firms receiving equity finance through selling c to the merger firms cover the costs of R&D projects and marketing activities through it.

Before deriving the merger firms' profit maximizing n, we state the following Propositions 1 and 2, for solving this problem.

Proposition 1. *m, $\tilde{m}$, and n have no effects on $f(t, \bar{j}(t), x(t)) = \log\left\{\frac{S(t)}{S(0)}\right\}$ where $f(t, \bar{j}(t), x(t))$ is the call-option price as in Black and Scholes ([8]).*

Proof. A proof of Proposition 1 is provided in the Appendix. □

The implication of this Proposition is that as soon as m, $\tilde{m}$, and n are determined, the effects on $S(t)$ and therefore on $S(0)$ are formulated so that the purchase of assets are arbitrage free. An economic rationale of this example is that as soon as the merger firm makes an announcement about n, the information of these start-up entrepreneurs would spread through the market and the stock price of the merger firm $S(0)$ would coincides $S(t)$, so that there would be no arbitrage. We can plausibly expect that as soon as the merger firm decides n, the announcement of it would be made and information of each n would be published. This is because large firm such as those that are listed on the New York Stock Exchange (NYSE) firms have many existing interim sources of information, for example, interim financial reports, trade journals, security analysts' forecasts, industry forecasts, litigation, prospectuses, etc ([16, p.255]).

Proposition 2. *The expected production function of the merger firms with respect to n is*

$$E\left[n\frac{\bar{j}(t)}{m}x(t)\right] = x(0)\exp\left\{(\bar{\mu} + \frac{mn}{\sigma^2})t\right\},$$

and is S-shaped when $Mt > 4\sigma^2$.

Proof. The proof of Proposition 2 is provided through simple calculations.[6]

[5] See Aghion and Howitt ([4, p.55]) for $n\frac{\bar{j}(t)}{m}$.

[6] See Elliott and Kopp ([15, p.124]) for reference, and employ Proposition 1 and Lemma 1 in the Appendix.

Proposition 2 depicts that initially, the merger firms hasten to take equity positions in other start-up entrepreneurial firms while attempting to acquire innovation. However, it is optimal for the merger firms to subsequently slow down their speed of acquisitions.

Using Proposition 2, we can derive the optimal number of start-up entrepreneurial firms that the merger firms take equity positions in when $t = T$ as[7]

$$n^* = -k - \frac{MkT}{2\sigma^2 W\left(\dfrac{-\exp\left[\frac{-\{(\bar{\mu}-r)\sigma^2+M\}T}{2\sigma^2}\right]}{2\sigma^2\sqrt{\frac{x(0)}{Mc\sigma^2 kT}}}\right)}, \tag{5}$$

where $\bar{\mu}$ is the drift of $\bar{j}(t)$ and W is the *Lambert W function*.[8]

The merger firms perform Open Innovation when $n^* > 1$. When $n^* = 1$, a firm conducts R&D on its own. When $n^* \leq 0$, the firms do not conduct any R&D projects, and thus do not purchase any call options.

In (5), k is the sum of the number of firms that the merger firms have no technological interest in and that of its rivals. On account of monopolistic competition, the merger firms disregard the behavior of other firms and thus the movement of k. Therefore n^* is unique. Because we are dealing with the new technology, the resulting market based on this product would only become monopoly or monopolistic competition. If the market becomes monopoly, i.e. $k = 0$, n^* would also becomes naught. Thus, in a world of monopoly, there is no Open Innovation. As an economic rationale of this proposition, for example, when MP3 player was first introduced into the market, the supplier of it did not have to worry about the behavior of other rivals that were selling other audio devices. But k affects n^* because they are also conducting R&D projects regarding audio devices. This is because the merger firms are trying to get the advantage of their rivals, and thus k has influence on their n^*. However, the technology used in MP3 player and other audio devices are different so the supplier of MP3 player disregards the behavior of other firms.

3 Simulations

We provide the numerical simulations of n^* below. We use Maple for these simulations.

In Fig.1, σ is in the range of 90 to 250, where c is normalized to 1, and $M = 10000$, $x(0) = 100$, $\bar{\mu} = 0.01$, $r = 0.1$, and $T = 3$ are substituted. The figure represents the cases when there is no observation noise related to consumer behavior. In the following subsection, we model and simulate the case wherein there is an observation noise.

3.1 Marketing with an observation noise

When consumer characteristics are not perfectly known or the marketing department observes noise related to consumer behavior whose characteristics are perfectly known,

[7] Use mathematical software such as Maple for this derivation.

[8] See Corless et al. ([11]) for the *Lambert W function*.

the Bayesian update founded on the normal distributions that is applied in (3) requires modification. According to Chamley ([9]), imperfect information on consumer behavior is operationally equivalent to observation noise related to consumer behavior whose characteristics are perfectly known.

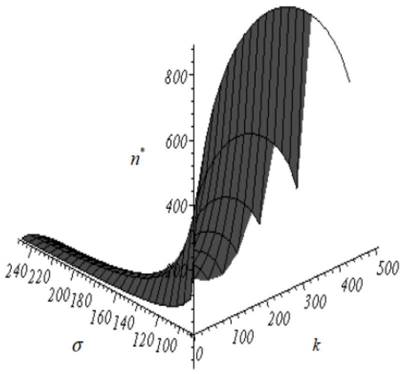

Fig. 1. Perfect information on consumer behavior.

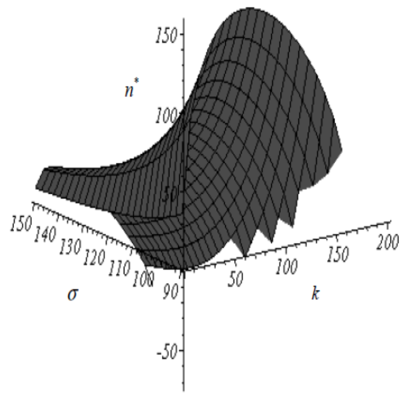

Fig. 2. Imperfect information on consumer behavior.

Hereafter, we modify the update of the equation derived from Bayes' Rule founded on the normal distributions as given in Chamley ([9, pp.48-50, (3.10)]) so as to cope with situations when consumer behavior is unknown. The precision of marketing activities based on the four Ps becomes different, and we remodel the stochastic differential equation expressing the incremental process of $x(t)$ as

$$dx(t) = \frac{\tilde{m}n}{\sigma^2 + \sigma_\eta^2(1+\rho\sigma^2)^2} x(t)dt + x(t)\sqrt{\frac{\sigma^2 + \sigma_\eta^2(1+\rho\sigma^2)^2}{n}} d\tilde{V}(t) \ , \qquad (6)$$

where ρ is the inverse of the variance of the prior distribution σ_θ^2 and σ_η^2 is the variance of an observation noise. Proposition 1 can also be applied to the production function that uses (6) and a proposition similar to Proposition 2 can be derived as follows.

Proposition 3. *When consumer behavior is not known, the expected production function of the merger firms with respect to n is*

$$E\left[n\frac{\bar{j}(t)}{m}x(t)\right] = x(0)\exp\left\{(\bar{\mu} + \frac{mn}{\sigma^2 + \sigma_\eta^2(1+\rho\sigma^2)^2})t\right\},$$

and is S-shaped when $Mt > 4[\sigma^2 + \sigma_\eta^2(1+\rho\sigma^2)^2]$.

Proof. The proof of Proposition 3 is similar to that of Proposition 2.

See Fig.10 for the examples of Proposition 3. Fig.10 illustrates the situation when $Mt < 4[\sigma^2 + \sigma_\eta^2(1+\rho\sigma^2)^2]$.

The derivation of n^*, in the case with an observation noise, is similar to the derivation of (5). Thus, n^* becomes

$$n^* = -k - \frac{MkT\sigma_\theta^4}{2\delta W\left(\dfrac{-\exp\left\{\frac{-\{(\bar{\mu}-r)\delta + M\sigma_\theta^4\}T}{2\delta}\right\}}{2\sqrt{\frac{x(0)\delta}{Mc\sigma_\theta^4 kT}}}\right)}, \tag{7}$$

where

$$\delta = \sigma^2\sigma_\theta^4 + \sigma_\eta^2\sigma_\theta^4 + 2\sigma_\eta^2\sigma_\theta^2\sigma^2 + \sigma_\eta^2\sigma^4 = \sigma^2\sigma_\theta^4 + \sigma_\eta^2(\sigma_\theta^2 + \sigma^2)^2.$$

Hereafter, we operate our simulations based on (7), where c is normalized to 1 and $M = 10000$, $x(0) = 100$, $k = 100$, $\bar{\mu} = 0.01$, $r = 0.1$, $T = 3$, $\sigma = 80$, $\sigma_\eta = 42.3$, $\sigma_\theta = 100$, and $C = 50$ are substituted except those arguments that are presented in the figures.

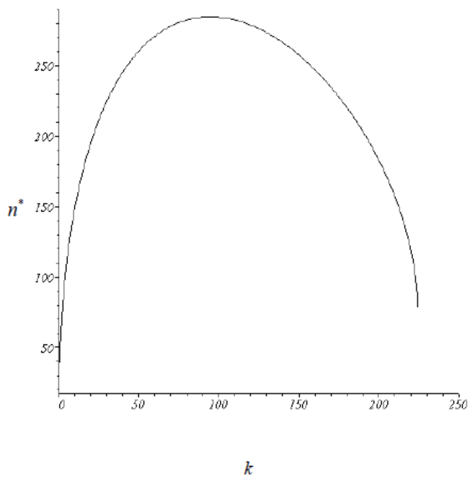

Fig. 3. k (*the sum of the number of firms that the merger firms has no technological interest in and the number of its rivals*) and n^* (*the optimal number of start-up entrepreneurial firms that the merger firms take equity positions in while attempting to acquire innovation*).

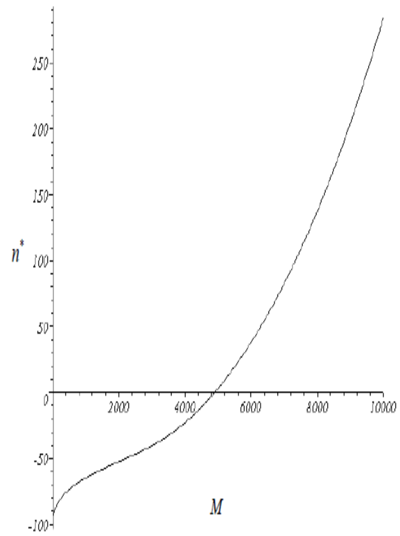

Fig. 4. M (*total number of R&D projects*) in the long term.

Fig.2 represents the version of Fig.1 that corresponds to the case when there is an observation noise related to consumer behavior. n^* is much smaller when the marketing department observes a noise than when it does not. As an economic rationale when merger firms are uncertain about the demand for their new technology, they reduce their investment in n.

In Fig.3, recognizing that the merger firms attempt to innovate significantly as k begins to increase, we can observe an incentive to "escape competition" ([1]). However, when the number of rivals in an industry is excessively high, the merger firms eventually desist from conducting R&D and taking equity positions in start-up entrepreneurial firms. This phenomenon is due to the "business stealing" effect [1, 2, 4]. In the words of Aghion and Howitt ([2]), rivals do not internalize the loss to the incumbents (the merger firms) by their entry. Considering the future loss evoked by the entries of rivals, the merger firms decrease n^*.

Fig.4 demonstrates the long term transition. It shows the transition in the long term from the stage of no innovation to the stage wherein the merger firms take equity positions in n while attempting to acquire innovation. When the number of researchers in an industry is limited and M is low, because of depression or scarcity of researchers in the field, the merger firms do not conduct R&D projects or do not take equity positions in other start-up entrepreneurial firms. Fig.4 shows that in order to take equity positions in other start-up entrepreneurial firms in attempting to acquire innovation, the merger firms require larger M and therefore an abundant number of researchers that engages in R&D.

Fig.5 reveals that the merger firms' performances are considerably better than that of non-merger firms. As shown here, according to the effective R&D projects and the augmented $x(t)$ through marketing activities, at the optimal point, the merger firms' profits are approximately 7 times larger than those of non-merger firms. We can observe that the difference between the profits of the merger and non-merger firms is "very sensitive [18]." Also we can see from "Tabel 5.7 Analysis of Merger Differences (scientific firms, 1966-77)" γ is almost 7 times higher comparing -0.03 for the "Restricted" sample and 0.65 for the "No-jump" period of the Merger firms, even though the laor productivity growth rates are equal for both [18, pp.116-117]. This conclusion would be supported by our results. The reason of this comes from well known fact that small firms have (relatively strong) merits in successful R&D projects and large firms have merits in marketing. Thus merger firms have the advantage of the two, and moreover, competitive behavior of researchers, since their internal labor market becomes competitive market for researchers do not know when the entry occurs by their firms' acquisitions.[9] Whereas by the *Closed Innovation* i.e. the opposite of Open Innovation, firm performs innovation by itself so that the boundary of the firm has a role as an entry barrier to the other researchers outside. By Open innovation, internal labor market of researchers becomes competitive market and this makes innovation cheaper.

And as an ex-post validity, Intel Capital held more than 475 investmenst in portfolio companies, with a market value of more than $1.4 billion ([10, p.126]). Cisco systems, from August 1998 to August 1999, acquired 14 companies with a total market

[9] See Shy [41, p.63] for example.

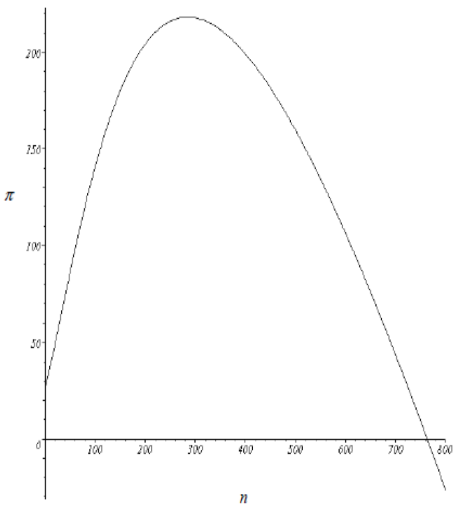

Fig. 5. π (*the merger firms' profits*) and n (*the number of start-up entrepreneurial firms that the merger firms take equity positions in while attempting to acquire innovation*).

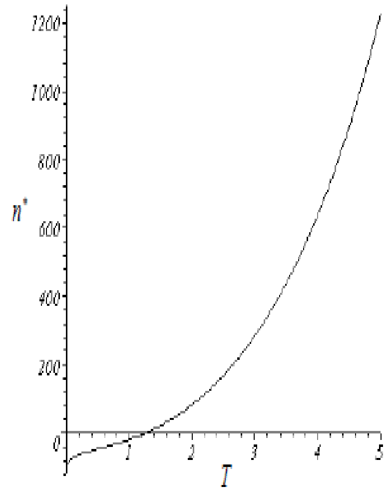

Fig. 6. T (*the maturity date of the call-option*) and n^*.

value of nearly \$12 billion ([36, pp.20-21, Exhibit 3]). In Sweden most successful small technology based firms are in due course acquired by large firms ([13, p.174]).

We have figures 6, 7, 8, 9, and 10 illustrate the relationships between T and n^*, σ and n^*, $\bar{\mu}$ and n^*, $x(0)$ and n^*, and n and $x(t)$ respectively.

4 Concluding remarks

The profit maximizing number of start-up entrepreneurial firms that the merger firms take equity positions in while attempting to acquire innovation is provided and simulated for each case when the marketing departments observe and do not observe a noise on consumer behavior. Further, the relationship between the number of start-up entrepreneurial firms that the merger firms take equity positions in while attempting to acquire innovation and profits excluding acquisition-related costs $x(t)$ is proven to be S-shaped whenever the number of an industry R&D projects are large enough to cover the variance of $x(t)$. We also observe that the merger firms' expected profits are revealed to be considerably higher than those of non-merger firms. What are the gains that increase the expected profits of the merger firms? Owing to the complete stock markets, the assets gained, including intangible and tangible assets (net assets), are arbitrage free. An acquisition per se does not result in gains or losses due to the no-arbitrage condition. Note that, in this model, invention by itself does not contribute to the profits or sales

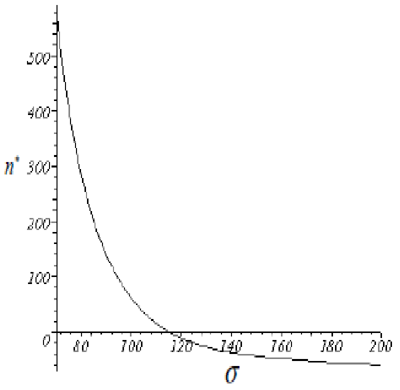

Fig. 7. σ (*derived from* σ^2 *of* $x(t)$) and n^*.

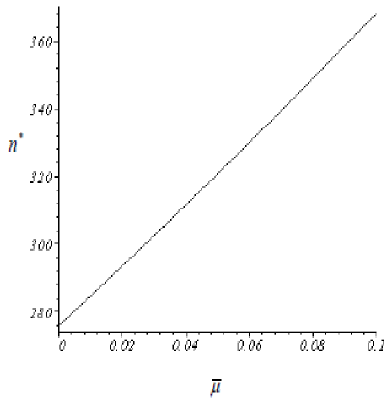

Fig. 8. $\bar{\mu}$ (*the drift of* $\bar{j}(t)$) and n^*.

of the merger firms. If the invented new technologies can not suffice the demands of consumers, they do not contribute to anything besides adding to the R&D costs. It is the integration of the marketing organized for the new technological products and the efficient R&D projects that correspond to the outcome of marketing activities during the period of R&D projects that raises $x(t)$ of the merger firms. The reason for this is well known fact that small firms have (relatively strong) merits in successful R&D projects and large firms have merits in marketing. Thus merger firms have the advantage of the two if these are combined frictionless, and moreover, competitive behavior of researchers, since their internal labor market becomes competitive market for researchers do not know when the entry occurs by their firms' acquisitions. By Open innovation, internal labor market of researchers becomes competitive market and this makes innovation cheaper.

Appendix

Derivation of (3). The variance of the equation derived from Bayes' Rule founded on the normal distributions after being updated t times is[10]

$$\frac{\sigma^2\sigma_\theta^2}{\sigma^2+\sigma_\theta^2 t}, \tag{A1}$$

where σ^2 is the variance of $x(t)$ and σ_θ^2 is the variance of the prior distribution. The inverse of this variance, the precision, is

$$\frac{\dfrac{\sigma^2}{n}+\dfrac{\sigma_\theta^2}{n}\tilde{m}t}{\dfrac{\sigma^2}{n}\dfrac{\sigma_\theta^2}{n}} \tag{A2}$$

[10] See Chamley ([9, p.25]) for example.

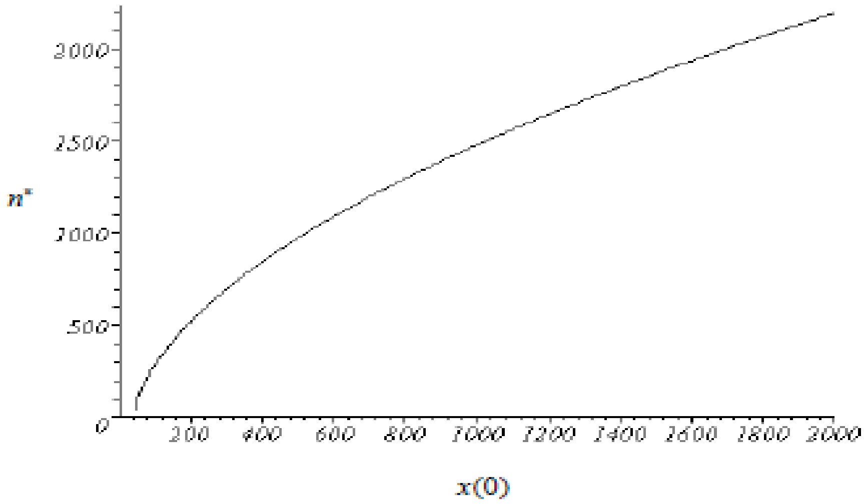

Fig. 9. $x(0)$ (*profits excluding acquisition-related costs in time 0*) and n^*.

where we replaced σ^2 by $\frac{\sigma^2}{n}$ and σ_θ^2 by $\frac{\sigma_\theta^2}{n}$ in order to take into consideration of the reductions in the variances by taking equity positions in n start-up entrepreneurial firms and evenly allocated $\tilde{m}$ for each i and t. $\frac{\sigma^2}{n}$ and $\frac{\sigma_\theta^2}{n}$ can be justified by assuming that the rational merger firms do not allow the start-up entrepreneurial firms that are receiving equity finance from them, to have the same prior distributions or to undertake the same marketing activities. Differentiating (A2) with respect to t yields

$$\frac{d\left[\dfrac{\dfrac{\sigma^2}{n}+\dfrac{\sigma_\theta^2}{n}\tilde{m}t}{\dfrac{\sigma^2}{n}\dfrac{\sigma_\theta^2}{n}}\right]}{dt} = \frac{\dfrac{\sigma_\theta^2\tilde{m}}{n}}{\dfrac{\sigma^2}{n}\dfrac{\sigma_\theta^2}{n}} = \frac{\tilde{m}n}{\sigma^2}\ . \qquad (A3)$$

We assume that this increment of precision per time to be the drift of $x(t)$ through marketing activities. Thus, a stochastic differential equation describing the incremental process of $x(t)$ is given by (3).

Proofs of the Existence and Uniqueness of Solutions of (2), (3), and (6). We first provide proofs of the existence and uniqueness of a solution of (2) in the range of $[0, T]$. The following proof is based on Klebaner ([23, pp.17-18]), Minotani ([35]), and Φksendal ([37, chap 5]). (2) satisfies the following condition:

$$|\mu(j,t)| + \left|\sqrt{\frac{j(j,t)}{n}}\right| \leq B(1+|j|);\ \ j \in \boldsymbol{R^n},\ \ t \in 0, T \qquad (A4)$$

for some constant B, and the following Lipschitz condition:

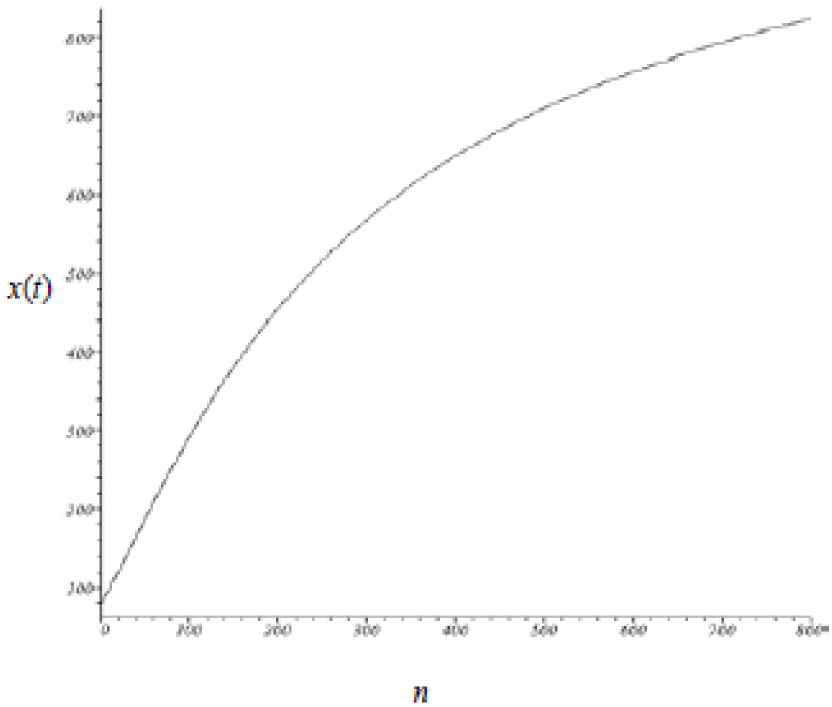

Fig. 10. n and $x(t)$ (*profits excluding acquisition-related costs of the merger firms*).

$$|\mu(j,t)-\mu(l,t)| + \left|\sqrt{\frac{j(j,t)}{n}} - \sqrt{\frac{j(l,t)}{n}}\right| \leq D|j-l|;\ \ j,l \in \boldsymbol{R}^{\boldsymbol{n}},\ \ t \in 0,T \qquad (A5)$$

for some constant D. Since μ is the drift of j, $\mu < 1$. Thus, (A4) is satisfied by an adequately large B, since j is the number of times of successful R&D projects. We prove the Lipschitz condition below. Here we assume that $\mu(j,t) = \mu_1 j(t) + b$ and $\sqrt{\frac{j(l,t)}{n}} = \sqrt{\frac{1}{n_1}}\sqrt{j(t)} + b_0$. Owing to $\mu < 1$ and an adequately large D, μ_1 and $\sqrt{\frac{1}{n_1}}$ satisfy the following conditions:

$$|\mu_1| < \frac{D}{2} \text{ and } \sqrt{\frac{1}{n_1}} < \frac{D}{2}\ . \qquad (A6)$$

Therefore,

$$|\mu(j,t)-\mu(l,t)| = |\mu_1\{j(t)-l(t)\}| = |\mu_1|\,|j(t)-l(t)| \leq \frac{D}{2}\,|j(t)-l(t)|\ , \qquad (A7)$$

and

$$\left| \sqrt{\frac{j(j,t)}{n}} - \sqrt{\frac{l(l,t)}{n}} \right| = \left| \sqrt{\frac{1}{n_1}} \sqrt{j(t)} - \sqrt{l(t)} \right| = \left| \sqrt{\frac{1}{n_1}} \right| \left| \sqrt{j(t)} - \sqrt{l(t)} \right| \leq$$

$$\leq \frac{D}{2} \left| \sqrt{j(t)} - \sqrt{l(t)} \right| , \qquad (A8)$$

Then the Lipschitz condition is satisfied. Consequently, the existence and uniqueness of a solution of (2) in the range of $[0, T]$ are proved, provided that the second moment of initial value $j(0)$ is

$$E[\{j(0)\}^2] < \infty . \qquad (A9)$$

Moreover, $j(0)$ must be independent of $V(t)$ and $t \geq 0$. These conditions are satisfied considering that $j(0)$ is the number of times of successful R&D projects in time 0 and is thus constant. □

Proofs of the existence and uniqueness of solutions of (3) and (6) in the range of $[0, T]$ are similar as to the above proof.

Proof of Proposition 1. Due to the Black-Scholes ([8]) economy, the call option price is

$$f(t, \bar{j}(t), x(t)) = \log \left\{ \frac{S(t)}{S(0)} \right\} = \log \left\{ \frac{R \times (n \frac{\bar{j}(t)}{m} x(t))}{Q \times (x(0))} \right\} , \qquad (A10)$$

where R and Q are the multipliers or price/earnings ratios (PERs). In (A10) earnings are profits and $nc + C$ are not subtracted from $n \frac{\bar{j}(t)}{m} x(t)$, since there are no gains or losses from the acquisition per se by the no-arbitrage condition. Thus, we can focus our study on the profits resulted from innovation and not from the arbitrage of the acquisition. The gained assets including intangible and tangible (net assets) are arbitrage free owing to complete stock markets of the financial sector. Thus, acquisition-related costs have no influence on $S(t)$. Hence, though $S(t)$ is a function of price/book-value ratio (PBR), PBR is not included in (A10) as PERs are. And without loss of generality, we can assume that the numbers of shares of the merger firms are constant through $[0, T]$. In the above equation, since R&D projects and marketing activities are conducted in $[0, T]$, m, $\tilde{m}$, and n are predetermined by the contract made before time 0. Due to complete stock markets, as soon as m, $\tilde{m}$, and n are determined the effects on $S(t)$ and therefore $S(0)$ are formulated so that the purchase of assets are arbitrage free. For simplicity, let $r = 0$. Owing to the Black-Scholes ([8]) economy, the stock pays no dividends or other distributions. Therefore, in this case, the forward price of $S(t)$ at time 0 equals $S(0)$. Since, the expected settlement amount of the forward, the forward price, and the future price coincide [14, chap. 8, pp.166-168] , $n \frac{\bar{j}(t)}{m} x(t)$ and $x(0)$ have the multipliers so that $S(t) = S(0)$ and thus, m, $\tilde{m}$, and n can not affect $f(t, \bar{j}(t), x(t))$. This is nothing but a Radner equilibrium [see Radner ([39])] under complete markets. Therefore, there are no effects of m, $\tilde{m}$, and n on $f(t, \bar{j}(t), x(t))$. That is,

$$f(t, \bar{j}(t), x(t), m, \tilde{m}, n) = f(t, \bar{j}(t), x(t)) . \qquad (A11)$$

□

Lemma 1. $E[\exp \int_0^t \sqrt{\frac{\bar{j}(s)}{n}} dV(s)] = \exp \int_0^t \frac{\bar{j}(s)}{2n} ds.$

Proof. First we prove that $\exp \int_0^t \sqrt{\frac{\bar{j}(s)}{n}} dV(s)$ is a martingale. We depend the following proof on Karatzas and Shreve ([22]). This is since,

$$Z_t(X) = 1 + \sum_i^d \int_0^t Z_s(X) X_s^{(i)} dW_s^{(i)}, \ \forall t \in [0, T]$$

which shows that $Z(X)$ is a continuous local martingale with $Z_0(X) = 1$. Solving for $Z_t(X)$ we get

$$Z_t(X) = \exp \int_0^t X_s dW_s,$$

where we applied $Z_0(0) = 1$ and omitted $\sum_{i=1}^d$ for d, dimension, is one in this case.

Thus, $J_t = \exp \int_0^t \sqrt{\frac{\bar{j}(s)}{n}} dV(s)$ is a continuous local martingale. If

$$E[\exp(\frac{1}{2} \int_0^T \|X_s|\|^2 ds)] < \infty; \ 0 \le T < \infty,$$

then $Z(X)$ is a martingale ([22, Section 3.5.D]) and therefore, J_t is also a martingale.

This is because, $\int_0^t \sqrt{\frac{\bar{j}(s)}{n}} dV(s)$ is bounded in t (since there is the maturity date) and $\bar{j}$ (since no researcher can attain successful R&D unboundedly and because of the lower bound g of μ). Martingale increments are independent of each other because, "*when squaring sums of martingale increments and taking the expectation, one can neglect the cross-product terms*[22, p.32]." Thus, applying $E[\exp \sqrt{\frac{\bar{j}(s)}{n}} V(s)] = \exp \frac{\bar{j}(s)}{2n} s$,

$$\begin{aligned} E[\exp \int_0^t \sqrt{\frac{\bar{j}(s)}{n}} dV(s) &= E[\exp\{\lim_{l \to \infty} \sum_{h=1}^l \sqrt{\frac{\bar{j}(\delta_h)}{n}} (V(s_h) - V(s_{h-1}))\}] \\ &= \lim_{l \to \infty} \prod_{h=1}^l E[\exp\{ \sqrt{\frac{\bar{j}(\delta_h)}{n}} (V(s_h) - V(s_{h-1}))\}] \\ &= \exp \int_0^t \frac{\bar{j}(s)}{2n} ds, \end{aligned}$$

where $\delta_h \in [s_h, s_{h-1}]$ and $max(s_l - s_{l-1}) \to 0$. In the above calculations we have employed that martingales are independent of each other, and thus, the expectations of multiplications are equal to the multiplications of expectations. □

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