The underwriting cycles under Solvency II[*]

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Abstract. The presence of the underwriting cycle in non-life insurance is well established. In recent years this subject has been renewed with the focus on the new European Directive, Solvency II, which supports the need to hold capital due to the uncertainty about the insurance premium. The standard formula used ignores the existence of two regimes, one where the premiums increase (a hard market) followed by an another where the premiums decrease (a soft market). We use a regime switching model to demonstrate the dynamic change of the variables and how it could be used in an internal model. Our analysis has been performed on the Motor Liability, Motor Damage and Property lines in the French market. The results support the main theories of the underwriting cycle and the need to allow a different behavior of the variables across the cycle. We suggest another specification for the premium risk in the standard formula

Keywords. Underwriting cycle, regime-switching, Markov chain.

M.S.C. classification. 91B30, 60J10, 62J02.
J.E.L. classification. G22, C31, C33.

1 Introduction

Since 1997 the European Commission and the Committee of European Insurance and Occupational Pension Supervisors (CEIOPS) have worked on the new rules of solvency for the European insurance industry. These rules, named Solvency II, have been implemented to improve risk management in the insurance industry by identifying different types of risks and allowing insurance companies to use an internal model to estimate their capital (see the article 110 of [5]). The different classes of risk proposed by the CEIOPS are: market risk, default risk, non-life risk, life risk, health risk and operational risk. Each of these risks are split into modules. For the non-life risk class, two types of risks are identified: underwriting risk and catastrophe risk. Following the definition, TS.XIII.A.1 in

[2]:"The underwriting risk is the specific risk arising from insurance contracts. It relates to the uncertainty about results of the insurer's underwriting". This risk concerns both premium and reserve risk.

The premium risk describes in the standard formula used volatility by line of business, an assumption of a lognormal distribution and the estimated earned premium to assess the need of capital. This approach disregards the underwriting cycle's behavior which can be separated in two regimes. A soft market is characterized by an extensive supply of insurance contracts and a decrease in the premium, and a hard market with a low supply of contracts and an increase in insurance premium.

Five main problems arise with the CEIOPS's methodology:

— The use of loss ratio (claims divided by the premium) to analyze the uncertainty about the premium, see TS.XIII.B.12 in [2]. A high loss ratio could be due to high claims or low premium, and a low premium could be due to a decrease in the number of policyholders or a premium's decrease. We cannot identify the cause of the loss ratio evolution.
— Using historic loss ratios based on nominal value, though the future rules of the European directive requires companies to estimate the losses in economic value.
— The CEIOPS applies the volatility estimated across the cycle, ie the volatility inter regime.
— The premium risk is applied without distinguishing the regime. In a hard market, the capital requirement would increase due to the premium increase.
— No investigation of the causes of the underwriting cycle.

To overcome these problems, we investigate the main theories in the fields of the underwriting cycle to model its behavior in an internal model and add some features to show that the required capital calculation in the standard formula results in an inverse output to the one originally planned.

— The capacity constraint hypothesis (supported by [11], [12], [13], [17], [22], [23], [24]) is based on the fact that an insurer has to hold an amount of capital to underwrite policies. In case of the occurrence of a shock which decreases the capital, an insurance company has to increase its premium to form its capital. Because it would be more expensive to raise capital through financial markets due to the information asymmetry.
— The financial pricing model is based on discounted cash flows. The insurance premiums are equal to policy expenses plus the expected value of future claims. Many papers have been written on this subject by [7], [14], [15], [19].
— In the actuarial model, the premium equals the expected present loss plus a risk loading, see [1].
— A time series analysis is performed on the behavior of the loss ratio to determine the evolution of the underwriting cycle, see [21].

We based our approach on the existence of two regimes (a hard and a soft market). It lead us to use a regime switching model in a Markovian environment.

Regression techniques are used to estimate the coefficient of the variables (loss, outstanding, lagged value of the premium evolution, surplus, investment income).

This analysis has been performed with different distribution functions of the regression to check the Solvency II 's assumption of the lognormal distribution. In order to test and estimate the regime switching model, we worked on a panel of 19 French insurance companies for the three main lines of business: Motor Liability, Motor Damage, Property. We can distinguish the volume and price effect on our panel from the premium earned by the insurer.

Our work shows the existence of different behavior for the variables in each regime (for example: lower volatility in the soft market than in the hard market, and the coefficient of the capital is significant in the hard market but not in the soft market). The assumption of a lognormal distribution for Motor Damage and Property is supported, but the normal distribution is more suitable for Motor Liability.

The Markovian environment used allows us to calculate the average length of the cycles. This length vary from 5.6 years for Motor Damage, 6 years for Motor Liability and 7.5 years for Property. The soft market is longer than the hard market for the Motor Liability lines, while the hard market is longer than the soft market for Motor Damage and Property Lines. This model can be easily implement in an internal model to derive the regulatory capital.

An alternative to the formula provide by the CEIOPS is suggested to better grasp the behavior of the underwriting cycle. Our proposition takes into account the regime and is in keeping with the methodology of Solvency II.

The remainder of this paper is organized as follows: The first section presents the hypothesis of the behavior of the underwriting cycle. Data, models and their outputs are then described in the next section. Finally, we summarize and conclude the study.

2   Framework of the underwriting cycle

2.1   French underwriting cycle

We consider three lines of business: Motor Damage, Motor Liability and Property for the non-life French insurance market from a panel of 19 companies, which represents 30% of the market. We focus on these three lines for different reasons. We need the number of contracts for these lines in order to distinguish between the volume and price effect of the premium earned. With this distinction, we are able to determine if the growth of the premium earned stems from an increase in the number of contracts or if it is due to the underwriting cycle. The earned premiums are scaled by this exposure.

Our panel is mainly constituted of mutual companies. They underwrite principally personal lines. These three lines account for approximately 60% of the gross earned premiums in the non-life market. In order to match the definition provided by the CEIOPS, the property line include personal, professional and agricultural property and natural catastrophe (specific treatment in France, see

[8] for a formal presentation). The period covered is from 1995 to 2006, when the data are available. The mean period observed from our panel is 8.2 years, in calculating the mean value for each year, we obtain the following output:
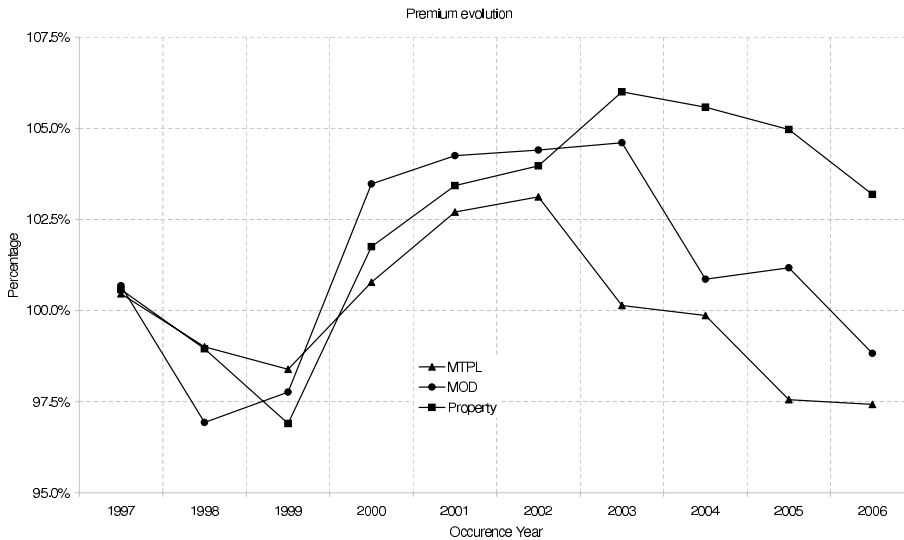
Fig. 1. Scaled Premium's Evolution for the Three Lines of Business

We clearly see a pattern of decrease followed by an increase.

The Motor Liability shows a decrease from 1997 to 1999, then an increase until 2003. A new period of decrease has occurred since 2003. Motor Damage shows a decrease until 1999, followed by a strong increase (more than 5pts) continuing until 2005. In 2006, a new decreasing trend appeared for this line.

The Property line shows a similar evolution (decreases until 1999, a strong increase in 2000. This increase sped up in 2003, then this increase slow down). The hard market observed for the property lines is relatively longer than the Motor Damage and Motor Liability one.

There are many reasons for this. We used the definition of the CEIOPS's line of business. As we mentioned, we aggregate different lines, it can introduce a bias in the length of the cycle as mentioned in [10].

Over the last decade, many natural events hit France:

- 1999: Two major windstorms
- 2002-2003: Several floods
- 2003: Drought
- 2006: Windstorm

These evolutions justify the need to hold an amount of capital in the soft market due to the underpricing, which increases the insolvency of insurance companies.

2.2   Underlying theories of the underwriting cycle

Underwriting cycle phenomenon is a major issue for the insurance industry when assessing the insurance pricing method and the regulation constraints.

Indeed a soft market may contribute to insolvencies if insurers price and underwrite too aggressively, while high prices and restrictive underwriting in the hard market may contribute to reduce the flows of goods and services.

No consensus appears in the literature. Many fields have been explored to obtain a stochastic model for the behavior of the premium.

Some analysis are based on the behavior of the loss ratio to determine the evolution of the underwriting cycle (see the papers of [7], [21]). They used time series analysis to show the cyclical pattern stem from regulatory and accounting lags.

Most attention has been focused on the financial pricing model and capacity constraint hypothesis.

The financial pricing models are based on discounted cash flows. The insurance premiums are equal to policy expenses plus the expected value of future claims. The insurance company is risk neutral and has rational expectation with respect to the claims. Many authors, such as [7], [14], [15], [19] use times series analysis to confirm financial pricing assumptions. This theory implies a positive coefficient for the claims outstanding, the paid losses and the premium's evolution lagged when regressing them with the premium's evolution. The coefficient of the interest rate has to be negatively linked with the premium's evolution.

The capacity constraint hypothesis is based on the fact that an insurer has to hold sufficient capital to meet its liabilities. In the case of an occurrence of a shock to the institution, an insurance company would have difficulties in raising capital in financial markets. The external capital would be more expensive than the internal one due to the information asymmetry between the manager and the investor that exists in financial markets. Regarding the increase of capital, investor does not see in a good perspective. If this were the case, it would increase its capital by self-financing. The investor would ask for a higher risk premium, so the company should increase its premium to form its capital. This approach involves a negative relationship of the capital's evolution between the premium's evolution. The capacity constraint is supported by [4], [11], [12], [13], [17], [22], [23], [24].

[6] proposed an extension to the capacity constraint hypothesis in adding the assumption that the policyholders agree to pay more if the insurer has a lower probability of default than its competitors.

The most commonly used pricing method is in practice the actuarial model. Premiums are equal to the expected present loss plus a risk loading. The risk loading is a buffer for having an acceptable ruin probability. This model is linked positively with the variance of losses and negatively on the capital. Actuarial model has been chosen by the CEIOPS to assess insurer solvency level, see [1].

All the previous papers, except [17], use the same statistical approach (regression, time series analysis) to model the underwriting cycle. Those techniques imply a time invariant parameter of the variables. It seems difficult to believe

that the variables have the same behavior across the cycle. For example, with the capacity constraint hypothesis, when the capital decreases, the premium should increase, but the explanation for premium decrease is more uncertain. The use of a regime switching model allows us to estimate the parameters of the variables in each phase of the cycle. We extend the work of [17] by allowing the standard deviation to be different in each state.

3   Empirical analysis

3.1   Model

We follow the approach proposed by [16] and [18]. We assume that the regime shifts are exogenous with respect to all realizations of the regression vector. We also assume that the exogenous variable is the realization of the two-states Markov chain with:

$$Pr(s_t = j | s_{t-1} = i, s_{t-2} = k, y_{nt-1}, y_{nt-2}, \ldots) = Pr(s_t = j | s_{t-1}) = P_{ij}. \quad (1)$$

Where

s_t is the variable representing the two state of the world.
y_{nt} is the premium's growth for the company n in year t.

The probability P_{11} is the probability to stay in the state 1, which is the state where an insurance company increases its premium. While P_{22} is the probability to stay in the state 2, which is the state where an insurance company decreases its premium. Since we have only two state and the Makov Chain is ergodic, then $P_{12} = 1 - P_{11}$ and $P_{21} = 1 - P_{22}$. We presume the situation of s_t only through the behavior of y_{nt}.

We define the information set as:

$$F_{t-1} = \left(y_{nt-1}, y_{nt-2}, \ldots, y_{(n-k)(t-1)}, y_{(n-k)(t-2)}, \ldots \right). \quad (2)$$

We use the following regression model for each line, based on panel data.

$$y_n^{S_t} = \beta_0^{S_t} + \alpha_n^{S_t} + \sum_{j=1}^{5} \beta_j^{S_t} x_{jn}^{S_t} + \epsilon^{S_t}. \quad (3)$$

for $n = 1, \ldots, 19$ and $s_t = 1, 2$.

$y_n^{S_t}$ is the premium growth for the firm n in the state s_t.
$\alpha_n^{S_t}$ is a constant specific to the company n in the state s_t.
$x_{jn^{s_t}}$ It represents the growth for the jth variable (interest rate, outstanding losses, losses paid, capital, lagged value of premium growth) for the company n in the state s_t.
$\beta_j^{S_t}$ are the regression coefficients to be estimated in the state s_t.
$\beta_0^{s_t}$ is a constant term in the state s_t.

ϵ^{S_t} is the error term in the state s_t.

We have :

$$\mu(y_n^{s_t}) = E(y_n^{s_t}|s_t = j, F_{t-1}) \tag{4}$$

$$= \beta_0^{S_t} + \alpha_n^{S_t} + \sum_{j=1}^{5} \beta_j^{S_t} x_{jn}^{S_t} \tag{5}$$

$$\sigma^2(y_n^{s_t}) = Var(y_n^{s_t}|s_t = j, F_{t-1}). \tag{6}$$

As s_t is exogeneous with respect to ϵ^{s_t}, we have:

$$E(\epsilon^{s_t}|s_t = j, F_{t-1}) = 0. \tag{7}$$

In the literature, the common assumption for the conditional distribution function of $(y_n^{s_t}|s_t = j, F_{t-1})$ is the Gaussian distribution.

The actual rules for determining the premium risk in Solvency II are based on the assumption that the underlying risk follows a lognormal distribution. To avoid restricting to these two cases, we investigate two other distributions: Gamma and Weibull.

We estimate the parameters by the maximum likelihood. The likelihood function is given by:

$$L = \prod_{n=1}^{N} \prod_{t=1}^{T} ln(f(y_n^{S_t}|F_{t-1})). \tag{8}$$

where

$$f(y_n^{S_t}|F_{t-1}) = \sum_{s_t} \sum_{s_{t-1}} f(y_n^{s_t}, s_t, s_{t-1}|F_{t-1})$$

$$= \sum_{s_t} \sum_{s_{t-1}} f(y_n^{s_t}|s_t, s_{t-1}, F_{t-1}) Pr(s_t, s_{t-1}|F_{t-1})$$

$$= \sum_{s_t} \sum_{s_{t-1}} f(y_n^{s_t}|s_t, s_{t-1}, F_{t-1}) Pr(s_t|s_{t-1}) Pr(s_{t-1}|F_{t-1}).$$

By definition $\sum_{i=1,2} Pr(s_{t-1} = i|F_{t-1}) = 1$.

We proceed iteratively to solve this algorithm via the recursive filter [16]:

$$Pr(s_{t-1} = i|F_{t-1}) = \sum_{s_{t-1}} Pr(s_t = i, s_{t-1}|F_{t-1})$$

$$= \sum_{s_{t-1}} \frac{f(y_n^{s_t}|s_t = i, s_{t-1}, F_{t-1}) Pr(s_t = i|s_{t-1}) Pr(s_{t-1}|F_{t-1})}{f(y_n^{s_t}|F_{t-1})}.$$

3.2 Data

We used the data from the financial reports of insurance companies published in French GAAP (nominal values). These data are similar to those used by rating agencies to estimate a rating based on public information, see [9], [20].

We focus on the three main lines of business for the French Market: Motor Liability, Motor Damage and Property. Our sample has been restricted to allow us to distinguish the premium's growth between the volume and price effect. Even if this panel is limited, our purpose is illustrative and we believe that if more data were available the results could be generalized. The data used in the regression are for each line:

Losses The losses occurring in the accident year and paid in the year
**Premium** The earned premiums in the accident year
Allocated investment income The investment income for each line of business
Outstanding The reserve of the losses in the calendar year (the sum of accident year outstanding plus the variation of previous year's reserve).
Capital The capital at the company level.

These variables are scaled by the number of contracts.

In the data used, no reference to interest rate appears, although it is present in the tested theories. In reviewing the results of the last two "Quantitative Impact Study" (QIS) carry out by the CEIOPS, the interest rate has an impact. The CEIOPS provided two different term structures (see the graph 2) to estimate the present value of losses.
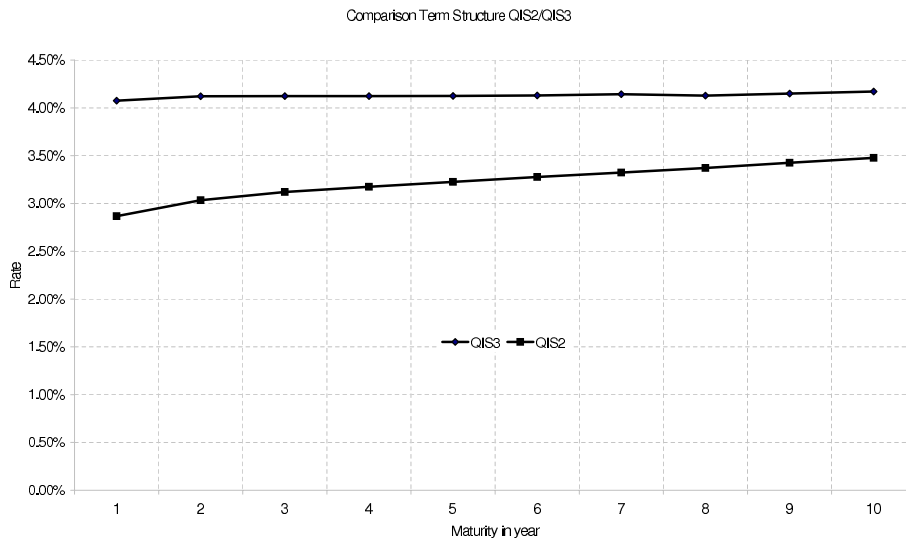
Fig. 2. Term Structure of the Second and Third Quantitative Impact Study

The following table presents the ratio of the expected discounted value of the outstanding losses with respect to the accounting value of the outstanding losses, for the French market for the second and third "Quantitative Impact Study".

Table 1. Ratio of the Discounted Value of the Cash Flow on the Accounting Value

	Best Estimate	
LoB	QIS2	QIS3
Motor (Damage + Liability)	85%	78%
Property	82%	81%

So with an increase of 30% in the interest rates, Motor (property + liability) line shows a decrease of 8% for the Best Estimate. For the Property line, no difference appears for the expected value. Clearly the difference in payment pattern appears between these lines.

Unfortunately we cannot specifically test the interest rate. Indeed, to estimate the discounted value of the cash flow of the outstanding losses, we need cash flow information on paid claims. It is not available. We could use the proxy defined by [24] but we should apply it for all the companies. It is difficult to believe that each company should have the same cash flow. The proxy works when we use times series, not panel data, as did [4], [24].

The same reason can be given for the use of interest rates in the explanative variables. Moreover a methodological problem arises with the use on the interest rate in the discounted value of cash flow and at the same time the use of the interest rate as the explanative variables.

Nevertheless, to verify the impact of the time value of the money, we include in the regression a proxy: The investment income allocated by line of business. We do this in order to see if a relationship appears with the premium, which should prove that the economic environment impacts the premium.

3.3 Results

Distribution function We test the four distributions, and we look at which gives us the maximum likelihood function.

Table 2. Results for the maximum likelihood function

Log Likelihood	Motor Liability	Motor Damage	Property
Gamma	136.72	171.43	141.78
Gaussian	*145.73*	181.4	150.34
Lognormal	134.51	*184.43*	*165.62*
Weibull	126.09	153.15	120.25

The lognormal distribution is the best distribution to represent the premium's growth for Motor Damage and Property. These results support the assumption of the CEIOPS, but we reject it for Motor Liability which exhibits a smaller tail than the lognormal.

As we work on panel data, we test for each line if the presence of the specific company constant (the parameter $\alpha_n^{S_t}$ in equation 3) increases the prediction capacity of the model by the AIC criteria. The AIC criteria is defined by:

$$AIC = \ln\left(\frac{\sum_{i=1}^{n} e_i^2}{n}\right) + \frac{2k}{n}$$

Where e_i is the residual of the regression, k is the number of regression parameters and n the size of our sample.

Table 3. AIC results

AIC	Motor Liability	Motor Damage	Property
With company effect	-6.14	-6.74	*-6.28*
Without company effect	*-6.63*	*-6.79*	-5.88

Specific company constant increases the performance of the model just for Property; for Motor Liability and Motor Damage, this effect can be ignored. One explanation for the presence of the specific company constant term in the Property line could stem from the fact that we include in the property line different types of risk (personal, agricultural, professional goods and natural catastrophic line). Even if the last three categories represent a small part of the earned premium it could affect the premium's evolution.

Regression estimation Following are the regression coefficients for each line, the standard errors are indicated in brackets:

Table 4. Regression Coefficients for the Motor Damage Line (*Significant at 5%)

Motor Damage	Soft Market	Hard Market
Constant	-3.09%* (0.91%)	3.52%* (1.51%)
Investment income	-0.22% (0.56%)	-0.35% (1.10%)
Outstanding	2.57%* (1.47%)	2.79%* (1.53%)
Losses	2.29%* (1.30%)	0.85% (3.41%)
Capital	1.26% (2.42%)	-4.40%* (1.40%)
Premium N-1	-0.72% (3.11%)	23.57%* (9.11%)
Volatility	2.02%* (1.03%)	3.47%* (1.87%)

All the variables have the expected signs, when they are significant. We observe that the claims paid and the outstanding losses are positively linked to the premium's growth. The coefficients of the outstanding losses are quite similar between the two phases of the cycle. We have a confirmation of the

Table 5. Regression Coefficients for the Motor Liability Line (*Significant at 5%)

Motor Liability	Soft Market	Hard Market
Constant	89.53%* (5.41%)	87.00%* (3.26%)
Investment income	-0.29% (0.50%)	0.06% (0.45%)
Outstanding	5.82%* (0.76%)	7.08%* (1.01%)
Losses	1.78%* (0.77%)	0.35% (0.62%)
Capital	1.64%* (0.84%)	-8.84%* (1.11%)
Premium N-1	0.49% (0.79%)	32.95%* (6.30%)
Volatility	3.19%* (1.76%)	4.76%* (2.21%)

Table 6. Regression Coefficients for the Property Line (*Significant at 5%)

Property	Soft Market	Hard Market
Constant	-5.41%* (2.90%)	5.90%* (2.49%)
Investment income	-0.21% (0.67%)	-0.06% (0.72%)
Outstanding	2.84%* (1.63%)	1.07% (1.33%)
Losses	-0.19% (1.14%)	2.04%* (1.22%)
Capital	1.32% (1.57%)	-5.26%* (2.98%)
Premium N-1	6.67%* (3.87%)	0.38% (1.88%)
Volatility	3.46%* (1.28%)	3.98%* (1.73%)

financial pricing theory and the actuarial model, because we have a premium adjustment to the claims. If we have an increase in the loss in year $n-1$, in year n we would have an increase in the premium.

We obtain a negative relationship between the capital and the premium's evolution in the hard market but the correlation is not significant in the soft market. As expected when the capital decreases, we observe an increase in the premium but not link appears between the premium's evolution and the capital in the soft market. This is a validation of the capacity constraint hypothesis for the hard market. This result confirms the use of a regime switching model.

As we could see, the coefficient of the investment income is negative, as expected, but not significant. Due to the large coefficient for the outstanding losses variable, a more accurate approach to treat the outstanding losses in a discounting value should be investigated to better reflect the impact of the economical environment.

We observe a strong correlation between the lagged value of the premium's evolution and the premium evolution in the hard market for the motor lines and in the soft market for the property line.

As the lagged value of the premium is not significant in soft market for the motor lines, it shows that when the loss decreases, the insurance companies reflect it on the insurance premium quickly. Inversely, we observe an inertia phenomenon in the hard market, we can conclude that due to the competition in the insurance market, the insurance company cannot reflect the whole increase of the loss in the premium, but it spreads it over many years.

We note an important difference in the volatility between the two regimes. For Motor Liability, in the soft market, the volatility is 40% lower than in the hard market. The same result can be observed on Motor Damage. The difference is smaller in Property but exists.

As there are frictional cost in the insurance market (tacit agreement, cancellation at renewal date) it is necessary for an insurance company to decrease its premium to avoid losing market share, which would be difficult to recover. It can explain why we observe a downgrade pressure on the premiums in the soft market.

The regime switching model allows us to see specifically in which phase of the cycle, the variables produce their effect on the premium's growth.

We perform the same analysis without distinguishing between the regimes. In doing a Chow Test, we obtain the following results:

Table 7. Chow test for the different lines of business

	P-value
Motor Liability	0.36%
Motor Damage	0.28%
Property	0.76%

As the P-value are below 1%, these results tell us that we have to distinguish between the regimes, and confirm the use of the regime switching model.

Transition matrix We present below the probability of the Markov Chain for 1-year.

Table 8. 1-Year Transition Matrix

Probability	Motor Liability	Motor Damage	Property
Probability of staying in hard market	61.66%	68.83%	42.42%
Probability of leaving hard market	38.34%	31.17%	57.58%
Probability of staying in soft market	70.67%	59.10%	82.52%
Probability of leaving soft market	29.33%	40.90%	17.48%

As we can see in the previous table, the probability of staying in the hard market for 1-year is higher than the probability of staying in the soft market for Motor damage. The reverse is true for the two other lines.

We have to note that the probability of staying in the hard market is two times lower than the probability of staying in the soft market for the Property lines. The aggregation of different types of risk can explain it.

We calculate the average length of a period in the soft market, ie a premium decreasing regime, and in the hard market, ie a premium increasing regime, with the Markov chain. Since the probability of having a period in regime i which lasts exactly t years is $p_{ii}^{t-1}p_{ij}$, we can calculate the average length of a period in regime i as :

$$\sum_{t=1}^{\infty} tp_{ii}^{t-1}p_{ij} = \frac{1}{p_{ij}}. \tag{9}$$

Table 9. Average Time in each Regime for each Line

Line of Business	Soft Market	Hard Market
Motor Liability	3.41	2.62
Motor Damage	2.45	3.21
Property	1.74	5.72

The length of a soft market is longer than the hard market for Motor liability, and the opposite for the other two. If we add the time spent in the soft market and the time spent in the hard market for each line, we obtain an average length of an underwriting cycle of 6.03 years for Motor Liability, 5.66 for Motor Damage and 7.46 for the Property lines. These results are quite similar to those obtain by [21] on US data. He found cycles of 6.3 years for Motor Liability (respectively, 5.5 for Motor Property and 7.2 for Property).

These results allows us to better grasp the behavior of the premium's growth and justify the need for an insurance company to develop an internal model which takes into account the underwriting cycle to determine the amount of capital for the premium risk and to better reflect the behavior of the insurance market. We wonder, if using a standard formula which integrates into the underwriting cycle we can achieve the same result.

Alternative model The previous model explains how to build a stochastic model to estimate the premium risk for the companies which have an internal model. As it is costly to develop an internal model for an insurance company (see [3]), we are interested in evaluating a standard formula which take into account the underwriting cycle.

As we mentioned in the introduction, the CEIOPS applies the volatility inter regime to the forthcoming earned premium which ever regime we are in, while it would be more realistic to apply the volatility intra regime and the premium's growth according to the regime.

On the graph 3, we can show the approach followed by the CEIOPS with the letters "A" and "B", they are applicable regardless of which cycle we are in. In the hard market (premium increase), the model of the CEIOPS requires more capital because the premium risk is calculated by means of a percentage

applied to the premium earned. But if the company increases its premiums, the insolvency risk should decrease.

In our alternative, we identify the regime. The distribution function represented by "C" has a smaller tail, representing the distribution function in a soft market. We apply the volatility specific to the soft market and the premium's growth.
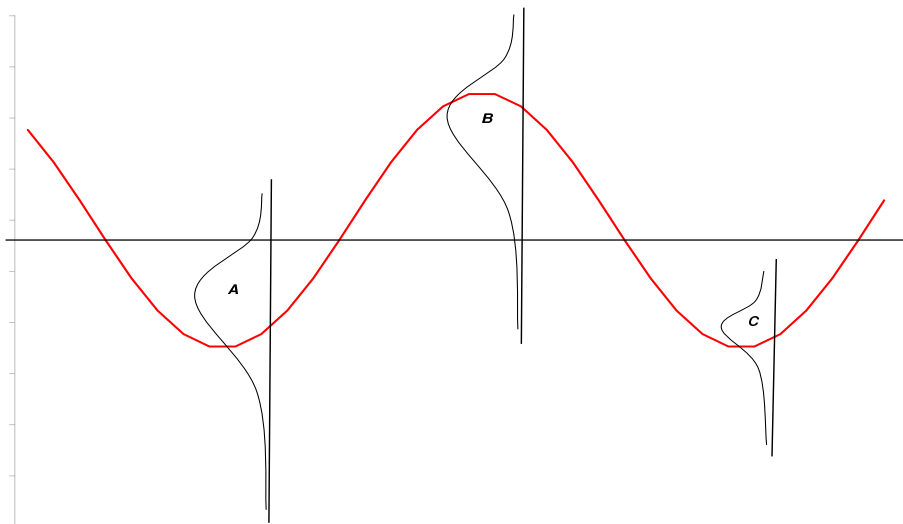
Fig. 3. Alternative model for the Premium Risk

More precisely, according to the current state of the cycle, we estimate the expected value for the premium risk:

$$PR = PE[\pi_{s_t,H}(\alpha_H + \rho(X_H)(1 + \alpha_H)) + \pi_{s_t,S}(\alpha_S + \rho(X_S)(1 + \alpha_S))]. \quad (10)$$

Where:

PR is the premium risk.
PE is the premium earned.
α_H, α_S represent, respectively, the premium's growth in the hard market and soft market (for example: 3% and -2%).
$\pi_{s_t,H}, \pi_{s_t,S}$ represent, respectively, the probability of moving from the state s_t to the hard market, and the probability of moving from the state s_t to the soft market. s_t is the current state, hard market or soft market.
$\rho(X_H), \rho(X_S)$ correspond, respectively, to the 99.5^{th} percentile of the distribution function (lognormal) in the hard market and the soft market.

These parameters would be estimated by the local regulatory authority.

Table 10. Parameters for the alternative model

Line	α_S	$\rho(X_S)$	α_H	$\rho(X_H)$
Motor Liability	-4.2%	-12.7%	4.8%	-9.3%
Motor Damage	-2.9%	-7.9%	4.5%	-5.3%
Property	-4.2%	-15.3%	5.5%	-6.3%

If we use our estimates (given in table 10) to calculate the premium risk with the same risk measure, we obtain the following percentages to apply to the earned premium:

Table 11. Comparison of the Premium Risk

Current State	Motor Liability	Motor Damage	Property
Soft Market	13%	7%	9%
Hard Market	9%	4%	4%
Solvency II	29%	29%	29%

Different explanations can be given to justify the spread.

– Using inter regime volatility, as CEIOPS does, rather than intra regime volatility, leads to a significantly higher capital requirement than in our model.
– The difference stems from the fact that the CEIOPS estimates the premium risk based on loss-ratios. While we have estimated the premium risk on premium scaled by the number of contracts.

In order to compare the impact of the underwriting cycle on the standard formula, we perform the same analysis using the loss ratio on two samples. One is the same as before (Panel A) and the other is an extended sample over 25 companies (Panel B) with a mean period of 9.4 years.

Table 12. Comparison of the Premium Risk with Loss Ratio

Sample	Current State	Motor Liability	Motor Damage	Property
Panel	Soft Market	22%	15%	30%
A	Hard Market	21%	6%	26%
	Solvency II	29%	29%	29%
Panel	Soft Market	22%	16%	30%
B	Hard Market	20%	6%	26%
	Solvency II	29%	29%	29%

We work on a calendar year basis. It means that the volatility and the growth in the loss ratio that we observe come from the uncertainty from the reserve and the premium. So our estimate corresponds to the premium risk and a part of the reserve risk. The inclusion of the underwriting cycle in the CEIOPS methodology could reduce the capital requirement and better reflect the behavior of the insurance market.

4 Conclusion

The CEIOPS recognizes the fluctuation of the insurance premium, and require insurance companies to hold an amount of capital to protect themselves against the uncertainty around the premium. Unfortunately, no consideration of the underwriting cycle is being made and nor is the current state of the cycle.

We tested the main theories of this subject on a sample of French companies. We extended them by allowing the variables to have a different behavior across the cycle. We used a regime switching model to integrate the two regimes of the underwriting cycle: a soft market characterized by an extensive supply of insurance contracts and a decrease in the premium; and a hard market with a low supply and an increase in the premium.

It is possible to isolate the theories which are applicable according to the regime (for example: negative link between the capital and the premium's evolution in the hard market only). Our work supported the capacity constraint hypothesis, the actuarial model and the pricing model theory. We show that assuming a constant volatility of the premium growth across the cycle, as CEIOPS does, is a misrepresentation of reality. The volatility of the underwriting cycle is different in each regime, the hard market has a stronger volatility than the soft market.

We suggest another approach to estimate the premium risk using the same methodology, which integrates the underwriting cycle and determines the capital requirement according to the regime.

The use of the loss ratios to estimate the premium risk rather than the use of the premium scaled, leads to an increase in the capital requirement because it includes other elements than the premium uncertainty (for example: loss uncertainty).

Further research is advisable on the impact of the introduction of historic economic loss ratio rather than nominal loss ratio for the estimate of the standard deviation to a certain whether it could create a pro-cyclicity or counter-cyclicity effect.

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