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Investment, Uncertainty, Caps, Taxes, Competition, Externalities, Welfare

JEL Codes
C61, D41, D62

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In a competitive industry where production entails a negative externality, a welfare-maximizing regulator considers, as control instruments, setting a cap on the industry output or levying an output tax. We embed this scenario within a dynamic setup where market demand is stochastic and market entry is irreversible. We firstly determine the industry equilibrium under each policy and then determine the cap level and the tax rate which maximize welfare in each case. We show that a first-best outcome can be achieved through the tax policy while the cap policy may only qualify as a second-best alternative.

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1. Introduction

This paper explores the regulation of market entries when the production of commodities exerts external costs. This may be the case when, for instance, the entry of foreign firms negatively affects the domestic industry (Koenig, 1985), further market entries increase pollution (Spulber, 1985) or the development of new properties harm residents by reducing open space (Anderson, 1993).

Regulation may affect industry structure and equilibrium. In this respect, a relevant strand of literature, investigating the impact of environmental policy, has shown, mostly using static models, that the internalization of the external cost generated by production depends on the degree of market competition. This is because the regulator must take into account the welfare losses that under imperfect competition may be due to distortions of the industry output and suboptimal market entries (see e.g. Spulber, 1985; Katsoulacos and Xepapadeas, 1995, 1996; Shaffer, 1995; Requate, 1997; Lee, 1999; Lahiri and Ono, 2007; Fujiwara, 2009; Lambertini et al., 2017; and the survey by Millimet et al., 2009). Another strand of literature has instead focused on the regulation of negative externalities in the framework of irreversible investment under uncertainty.[1] Baldursson and von der Fehr (1999) study the efficacy of price and quantity controls in a setup where the investment in abatement is irreversible for some firms in the industry and reversible for others. The uncertainty in their analysis springs from the number of the firms using reversible abatement technology which is assumed to be stochastic. Their main finding is that the relative slope of the cost curve with respect to the slope of the benefit curve determines whether tax or cap is the better

[1]Chao and Wilson (1993) show that the option value affects the investment in abatement under uncertain permit prices. Xepapadeas (1999) studies how a firm respond to environmental policy when deciding their investment in abatement and location under uncertainty about the output price, the policy context and the technology. Zhao (2003) shows that when considering uncertainty about the abatement costs the magnitude of the option value is larger when introducing taxes rather than permits.

policy. Jou and Lee (2008) consider a real estate market assuming that newly developed properties, by reducing open space, have an external cost. They find that the internalization of such cost requires setting a positive tax on development or a negative tax on land values. In a similar framework, Lee and Jou (2007) show how the regulator can correct the negative externality by imposing a density ceiling control. Di Corato and Maoz (2019) search for the optimal cap on private firms' entries in markets where production has adverse externalities and the output price is stochastic.[2] As the literature on investment under uncertainty shows, in such a case the price which triggers market entries is above the marginal private cost as it also includes an "uncertainty premium" (see Dixit and Pindyck, 1994). They show that the higher the price uncertainty the higher are this uncertainty premium and the price threshold triggering market entries, implying that a greater part of the externality (possibly all of it) is taken care of by the endogenous actions of firms, reducing thus the impact on welfare of the external damage.

In this paper, we set up a model analyzing the industry equilibrium under perfect competition in a dynamic setup where market demand is stochastic and entry is irreversible. Production generates an external cost for Society which is assumed increasing and convex in the industry output. We then consider the following two polar policy instruments for regulating the negative externality: (i) a quantity control exerted by introducing a cap on the industry output and then rationing market entries; (ii) a price control exerted by imposing an output tax. We characterize the industry equilibrium under each policy and try to find which of the two policy tools leads to greater welfare.

[2]The model builds on Bartolini (1995) where, differently, no external damages associated with firms' investment and production are explicitly considered and the cap is taken as exogenous.

Our main findings are as follows. In the case of a cap policy we find that optimality leads to on an internal welfare-maximizing cap level. Rather intuitively, this level is the quantity at which the marginal market surplus is equal to the marginal social cost, i.e., the sum of private and external costs. If the current market quantity is still below this level, then it is optimal to set this level as a cap on market quantity and allow the market to expands towards it over time, based on the strategic entry considerations of the firms. If, on the other hand, the current market quantity is already above this welfare-maximizing cap level, then, due to irreversibility, the market cannot revert to this welfare-maximizing cap level, and it is optimal to set the cap at the current level, i.e., to immediately ban any further entry. We also find that the welfare-maximizing cap level is increasing in the level of market uncertainty, which implies that greater profit uncertainty makes the policy maker allow more entries. This result is based on the same effect by which the uncertainty premium counterbalances the external cost in Di Corato and Maoz (2019).

The result that the optimal policy in this case is based on an internal welfare-maximizing cap level is novel because the only other study searching for an optimal cap within a competitive environment with profit uncertainty and investment irreversibility is Di Corato and Maoz (2019) which reaches a different result. Specifically, they assume that the external cost is a linear function of the quantity produced, and therefore reach the result that it optimal to either have no cap at all, if the uncertainty is high enough, and otherwise to set the cap at the current market quantity. In contrast, in this study we assume that the external cost is a convex function of market quantity, as the empirical literature about pollution damages often suggests, and therefore reach the result of an internal welfare-maximizing cap level.

In the case of a tax policy we show that the output tax can be viewed as an additional cost of production for the private firm and its impact can be studied using the model by Leahy (1993). In his model, the price threshold triggering market entries is increasing in the cost of production, therefore, the introduction of an output tax, by raising the entry threshold, delays market entries with respect to the scenario where the industry is not regulated. This is because the output price, in its random evolution, needs more time (in expected terms) before hitting eventually a higher threshold. We then determine the tax rate maximizing welfare and find that it must be set equal to the marginal external cost associated with the industry output supplied at each time period. This implies that further market entries become less and less likely as the industry output increases since the higher the tax burden, the higher the entry threshold.

Finally, when comparing the cap and the tax policies, a relevant trade-off emerges. With the cap, the industry output is bounded but the cap does not affect its temporal evolution with respect to the scenario where the industry is not regulated. In contrast, with the output tax, there is no limit to market entries but the tax affects the temporal evolution of the industry output by delaying market entries.

We then show that a first-best outcome can be achieved only by adopting a tax policy and that, in this respect, the ability to affect the entry timing is crucial. In fact, by setting the tax rate equal to the marginal external cost associated to the industry output supplied at each time point, the externality is fully internalized by the firms and consequently entries occur only when the exogenous stochastic shifts in market demand yield an associated gain in terms of market surplus covering the marginal social cost of an additional unit of the good. In contrast, a cap policy may only be considered as a second-best alternative since, in the presence of a cap, market entries occur at a socially suboptimal time. In fact, when the cap is not binding, firms keep entering the market using the same strategy that would be followed in the absence of regulation while, when the cap is binding, market entries do not occur at all, even when they would be beneficial from a welfare perspective.

The paper remainder is as follows. In Section 2, we present our model set-up. In Section 3, we determine the industry equilibrium under no policy intervention. In Section 4, we introduce the two policy instruments for externality control and determine the optimal entry strategy under each policy. We determine the optimal cap and the optimal tax rate, compare the two polices and discuss our findings. In Section 5, we provide some remarks on our results and conclude.

2. The basic model

Within a continuous time setting, we consider a competitive industry comprised of a large number of identical firms that producing a certain good. Their individual size, *dn*, is infinitesimally small with respect to the market and they are all price takers.[3]

At each time point $t \geq 0$, the demand for this good is given by:

(1) $$P_t = X_t \cdot \phi(Q_t),$$

where P_t and Q_t are the market price and quantity of the good, respectively, $\phi(Q_t)$ is a deterministic component of the market demand with $\phi(Q_t) > 0$ and $\phi'(Q_t) < 0$ for any $Q_t > 0$, and $\lim_{Q_t \to \infty} \phi(Q_t) = 0$. The term X_t, is a demand shift factor that evolves stochastically over time according to the following Geometric Brownian Motion:

[3] Assuming that firms are of infinitesimally small size is standard in models investigating the competitive equilibrium in a dynamic setting. See for instance Jovanovic (1982), Hopenhayn (1992), Lambson (1992), Leahy (1993), Dixit and Pindyck (1994, Ch. 8), Bartolini (1993, 1995) and Moretto (2008).

(2) $$dX_t = \mu \cdot X_t \cdot dt + \sigma \cdot X_t \cdot dZ_t ,$$

where μ is the drift parameter, σ is the instantaneous volatility, and dZ_t is the increment of a standard Wiener process satisfying $E(dZ_t) = 0, \quad E(dZ_t)^2 = dt$ at each t.

Each firm rationally forecasts the future evolution of the whole market. Market entry is free and an idle firm can enter the market at any time. By entering the market, the firm commits to permanently offer one unit of the good at each t. This implies that the industry output, Q_t, equals the number of active firms in the industry. Producing one unit of the good has a cost equal to $M > 0$.

Production entails a negative externality that firms do not incur. Its cost for Society, $D(Q_t)$, is a function of the industry output Q_t. We take the standard assumptions that $D'(Q_t) > 0$ and $D''(Q_t) > 0$ for any $Q_t > 0$ and $D(0) = 0$, implying that the external cost is positive, increasing and convex in the industry output.

Last, firms are risk-neutral profit maximizers and discount future payoffs using the interest rate r.[4] As standard in the literature, we assume that $r > \mu$ to secure that the firm's value is finite.

3. Industry equilibrium under no policy intervention

Let start by considering a scenario where no control policies are present. Under our model setup, a firm contemplating market entry is facing the same situation as the

[4] Note that introducing risk aversion would not affect our results, but merely require the development of the analysis under a risk-neutral probability measure. See Cox and Ross (1976) for further details.

investors in Leahy (1993). Therefore, in the following, we use Leahy's analysis in order to determine the optimal entry strategy.[5]

At each time t, an idle firm has to decide whether to enter the market or not. By assumption, a firm entering the market commits to permanently produce one unit of the good at a cost equal to M. The present value of the associated flow of production costs, i.e. M/r, can be viewed as the irreversible investment that a firm must undertake in order to enter the market. As future revenues are uncertain, market entry will occur when the expected profitability of such investment is sufficiently high.

Let $V(X,Q)$ be the value of an active firm given the current levels of X and Q. The standard no-arbitrage analysis in Appendix A shows that

(3) $$V(X,Q) = Y(Q) \cdot X^{\beta} + \frac{P(X,Q)}{r-\mu} - \frac{M}{r},$$

where $\beta > 1$ is the positive root of the quadratic equation

(3.1) $$\tfrac{1}{2} \cdot \sigma^2 \cdot x^2 + \left(\mu - \tfrac{1}{2} \cdot \sigma^2\right) \cdot x - r = 0 .$$

In (3), the term $\frac{P(X,Q)}{r-\mu} - \frac{M}{r}$ represents the expected present value of the flow of the firm's future profits conditional on Q remaining forever at its current level. Therefore, the first term, $Y(Q) \cdot X^{\beta}$, accounts for how future market entries reduce the value of the firm, as the firm’s profit falls when the industry output Q increases.

[5] In the following, we will drop the time subscript for notational convenience.

Two boundary conditions are required for finding the threshold function $X^*(Q)$ triggering market entry. The first one is the *Value Matching Condition*:

$$(4) \qquad V\left[X^*(Q), Q\right] = 0 \ ,$$

and the second one is the *Smooth Pasting Condition*:

$$(5) \qquad V_X\left[X^*(Q), Q\right] = 0 \ .$$

Condition (4) is a standard zero-profit condition at the entry requiring that the value of an idle firm, which is null under free entry[6], must equal the value of an active one. Condition (5), in contrast, is an optimality condition that concerns the evolution of the demand shift, X_t, over time. Each time the process $\{X_t, t \geq 0\}$ hits the threshold $X^*(Q)$ a new firm enters the market and the price of the good, $P(Q)$, lowers since the supplied market quantity output has increased (see Dixit and Pindyck, 1994, Ch. 8, pp. 252-260). Thus, $X^*(Q)$ is an upper reflecting barrier regulating the process $\{X_t, t \geq 0\}$ by keeping its level over time below $X^*(Q)$.

Solving the system [4-5] yields the following result:

Proposition 1*:* Entry in a perfectly competitive market occurs every time the process $\{X_t, t \geq 0\}$ hits the threshold:

[6] The option to wait is valueless under free-entry since, as entry is attractive also for other firms, the firm, by postponing its entry, may lose out an investment opportunity (see Dixit and Pindyck, 1994, Ch. 8, pp. 256-258).

(6) $$X^{*}(Q)=\frac{\hat{\beta}\cdot(r-\mu)\cdot\frac{M}{r}}{\phi(Q)},$$

where $\hat{\beta}\equiv 1+\frac{1}{\beta-1}>1$.

Proof: Follows from applying (3) in (4) and (5). □

From $\phi'(Q_t)<0$ it follows that the threshold $X^*(Q)$ is an increasing function of Q, implying that the larger the market quantity supplied, the stronger the competition and then, ceteris paribus, the higher the profitability required for entering the market. Figure 1 schematically shows the entry dynamics based on the threshold $X^*(Q)$.

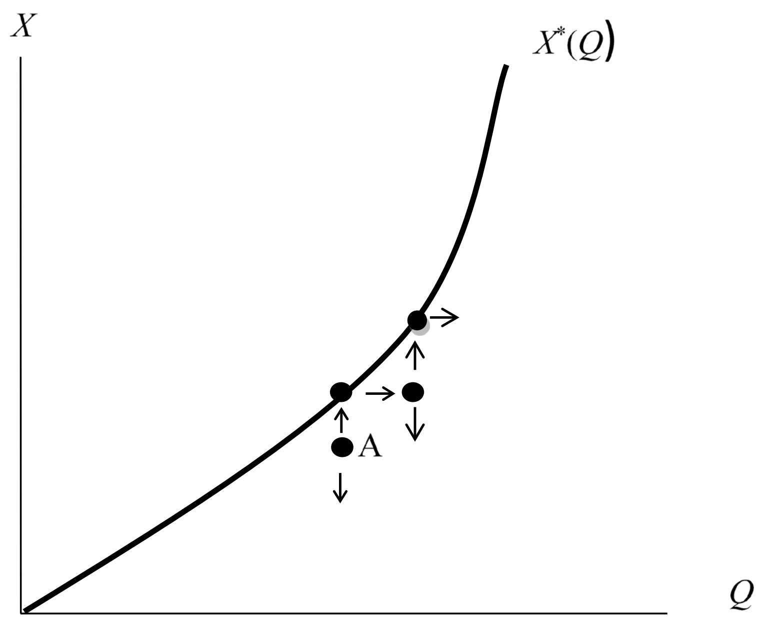

Figure 1: Demand swings and entry dynamics in a competitive industry. When the market is at a point like A, below the entry threshold, the swings in the demand shit factor, X, do not change market quantity. When X hits the threshold function $X^*(Q)$, firm entry leads to an incremental increase in Q making X once again below the threshold line.

In time intervals where Q is not changed, the changes in X are translated, via (1), to changes in P. Based on standard properties of Brownian Motions, in such time intervals the proportional connection between X and P, as captured by (1), implies that P is also a Geometric Brownian Motion, and with the same parameters as X. On the other hand, at time instants when X hits the threshold function $X^*(Q)$, then a rise in X is not translated into a rise in P but leads to an increase in Q which keeps P unchanged. This occurs at the following level of P:

(7) $$P^* = X^*(Q) \cdot \phi(Q) = \hat{\beta} \cdot (r - \mu) \cdot \frac{M}{r},$$

Which makes P^* an upper reflecting barrier regulating the process $\{P_t, t \geq 0\}$ and preventing the price from going above the level P^*. Figure 2 provides an illustration of these dynamics.

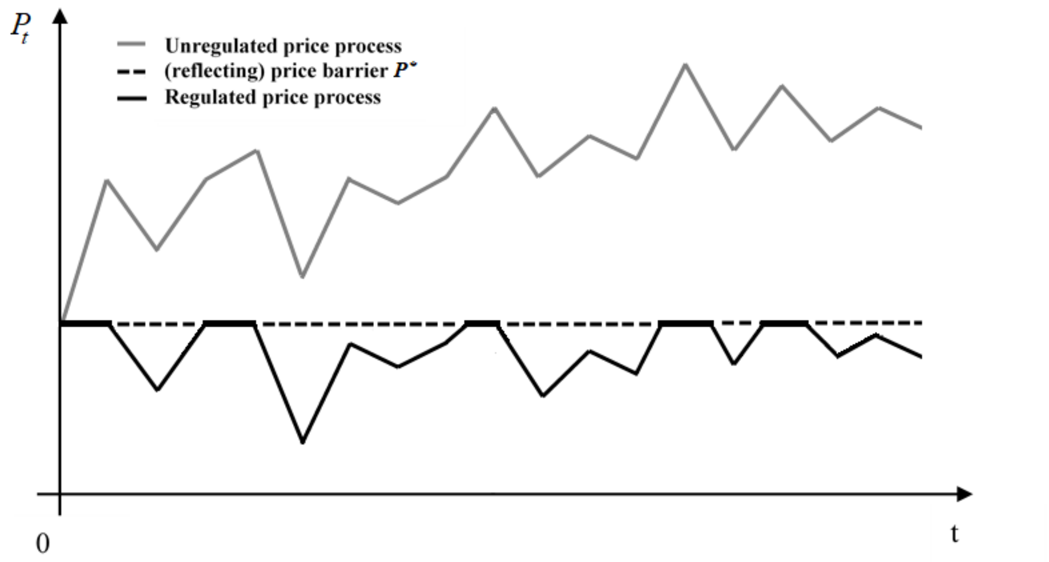

Figure 2: Price dynamics in a competitive industry

Note that, by the Marshallian rule, a firm should enter the market as long as $P = X \cdot \phi(Q) \geq (r - \mu) \cdot \frac{M}{r}$. Hence, by (6), the term $\frac{1}{\beta - 1} > 0$ in $\hat{\beta}$ is the wedge by which the entry threshold should be adjusted in order to take the uncertainty and irreversibility into account (see Dixit and Pindyck, 1994, Ch. 5, Section 2). Last, note that $\frac{dP^*}{d\sigma^2} > 0$ which follows from (7) taken together with the definition of $\hat{\beta}$ and also with $d\beta / d\sigma < 0$ which is established in appendix A. This means that the higher the demand volatility, the higher the price threshold triggering firm's entry, which implies that market entry is delayed. This is because the output price, in its random evolution, needs more time (in expected terms) before hitting eventually a higher threshold.

4. Industry equilibrium under policy intervention

The optimal entry strategy based on Eq. (6) does not account for the external cost associated with the negative externality that production entails once the firm has entered the market. In this section, we consider two policies for the reduction of the external cost: i) a cap on the industry output and ii) a tax on each unit of output. We first determine the industry equilibrium under each policy and then the level of the cap and the tax rate, respectively, maximizing welfare.

4.1 Industry equilibrium and welfare under a cap on the industry output

Assume that the regulator sets a cap on the aggregate industry output. Further, assume that entry licenses are distributed when the cap is announced. Each license allows producing one unit of output and their number is equal to difference between the cap, $\overline{Q}$, and the current level of the aggregate industry output, Q. We abstract from how the licenses are distributed since for our purposes their distribution has no other implications than providing to each firm owning a license the right to enter the market.[7]

[7] Note that, as shown by Bartolini (1995), the government may fully extract the producer's surplus by allocating licenses through a competitive auction.

4.1.1 The optimal entry strategy

The analysis of the firm's optimal entry under rationing is technically similar to the analysis in Section 3. The relevant difference between the two cases is that in this case the option to enter is an asset having a positive value that the firm gives up by entering the market. Thus, alongside the function $V(X,Q)$ which represents the value of an active firm, we define the function $F(X,Q)$ which stands for the value of the option to enter the market. A standard no-arbitrage analysis, similar to the one conducted in Appendix A for determining the value of an active firm, yields:

(8) $$F(X,Q) = H(Q)\cdot X^{\beta},$$

(9) $$V(X,Q) = Y(Q)\cdot X^{\beta} + \frac{P(X,Q)}{r-\mu} - \frac{M}{r},$$

where $H(Q)$ is to be found alongside the threshold $X^*(Q)$ by imposing the following *Value Matching Condition*:

(10) $$V\left[X^*(Q),Q\right] = F\left[X^*(Q),Q\right],$$

and *Smooth Pasting condition*:

(11) $$V_X\left[X^*(Q),Q\right] = F_X\left[X^*(Q),Q\right].$$

Condition (10) asserts that the value of the option to enter, that is, the implicit cost of market entry, equals the value of an active firm, that is, the implicit return associated with market entry. Condition (11) secures optimality by imposing that the marginal cost of market entry equals its marginal return. As shown by Dixit (1993), Condition (10) holds for any entry threshold and merely reflects a no arbitrage assumption, while Condition (11) is an optimality condition which holds only at the optimal threshold.

Proposition 2*:* In a perfectly competitive market with cap on the aggregate industry output, as long as the quantity in the market, *Q*, is below the cap, new entries to the market occur every time the process $\{X_t, t \geq 0\}$ hits the threshold:

$$X^*(Q) = \frac{\hat{\beta} \cdot (r - \mu) \cdot \frac{M}{r}}{\phi(Q)}, \tag{12}$$

or, equivalently, when the process $\{P_t, t \geq 0\}$ hits the barrier P^*, as captured by (7).

Proof: Follows from applying (8) and (9) in (10) and (11). □

Notably, the threshold function (12) does not depend on $\overline{Q}$ and is equal to the threshold function (6) determined under no policy intervention. The relevant difference here is that $X^*(Q)$ applies only until the cap $\overline{Q}$ is reached.

By *Proposition 2* and (7), a new firm enters the market every time the process $\{P_t, t \geq 0\}$ hits the upper reflecting barrier P^*. As explained above, this prevents the price from going above the level P^*. However, under a cap policy, the regulation of the price through the barrier control applies only until the cap $\overline{Q}$ is reached and, once there, the output price starts moving freely over time following only the evolution dictated by (2). Figure 3 provides an illustration of these dynamics.

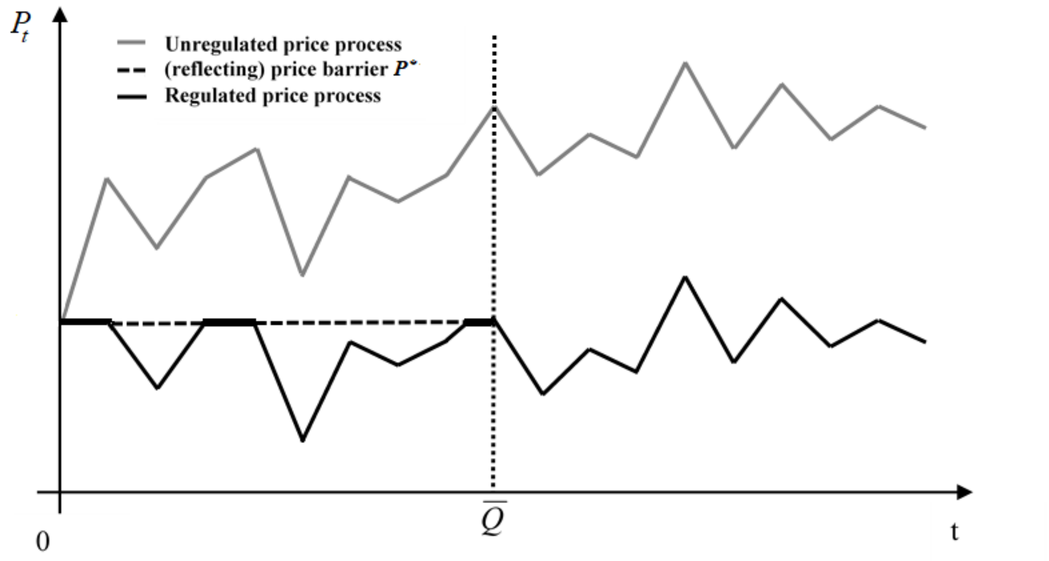

Figure 3: Price dynamics under a cap on the industry output

4.1.2 Welfare and the optimal cap

Once determined the industry equilibrium, in this section we determine the cap level maximizing welfare. This optimal level will trade off the welfare gains associated with lower negative externalities and the losses, in terms of market surplus, due to a lower quantity of the good available on the market once the cap has been reached.

Following a procedure similar to the one conducted in Appendix A for determining the value of an active firm, the expected discounted social welfare, given the current levels of X, and Q and the cap set at $\overline{Q}$, is:

(13) $$W\left(X,Q,\bar{Q}\right)=C\left(Q,\bar{Q}\right)\cdot X^{\beta}+\int_{0}^{Q}\left[\frac{P\left(X,q\right)}{r-\mu}-\frac{M+D'\left(q\right)}{r}\right]\cdot dq\,,$$

The integral in (13) represents the expected present value of welfare if the current industry output level, *Q*, will never change. For each unit supplied, the term $\frac{P(X,q)}{r-\mu}$ is the expected present value of the flow of market surplus associated with the supply of each unit of the good whereas the term $\frac{M+D'(q)}{r}$ is the present value of the flow of social costs associated to its production, i.e. private production costs plus external costs. The first term, $C\left(Q,\bar{Q}\right)\cdot X^{\beta}$, captures instead the contribution of future market entries to welfare.

At $X^{*}(Q)$ the following *Value Matching Condition* must hold:

(14) $$W_{Q}\left[X^{*}(Q),Q,\bar{Q}\right]=0\,.$$

Condition (14) is a standard boundary condition stating that each market entry raises welfare by $\frac{P\left[X^{*}(Q),Q\right]}{r-\mu}-\frac{M+D'(Q)}{r}$ via the supply of an additional unit of the good, but at the same time it also lowers welfare by $C_{Q}\left(Q,\bar{Q}\right)\cdot X^{*}\left(Q\right)^{\beta}$ in that the forgone market entry lowers the value of the contribution to welfare by future market entries.

Further, at $Q=\bar{Q}$ we must impose that:

(15) $$C\left(Q,\bar{Q}\right)=0\,.$$

The intuition behind Condition (15) is that the term $C(Q,\bar{Q})\cdot X^{\beta}$ in (12) captures the welfare associated with future entries to the market. No such changes are possible once Q has reached the cap $\bar{Q}$ and thus $C(Q,\bar{Q})$ must be null at $Q=\bar{Q}$.

Based on (13), (14) and (15) we show in Appendix B that:

(16) $$C(Q,\bar{Q})=\int_{Q}^{\bar{Q}}\left[\frac{P^{*}}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot\frac{1}{X^{*}(q)^{\beta}}\cdot dq .$$

Differentiating $C(Q,\bar{Q})$ with respect to $\bar{Q}$ yields:

(17) $$C_{\bar{Q}}(Q,\bar{Q})=\left[\frac{P^{*}}{r-\mu}-\frac{M+D'(\bar{Q})}{r}\right]\cdot\frac{1}{X^{*}(\bar{Q})^{\beta}} .$$

Eq. (17) leads to the following Proposition:

***Proposition 3**:*

(a) If the current industry output level, Q, is sufficiently large so that $\frac{P^{*}}{r-\mu}\leq\frac{M+D'(Q)}{r}$ then it is optimal to set the cap at the current Q, i.e., to immediately ban any further market entry.

(b) otherwise, if the current industry output level, Q, is sufficiently small so that $\frac{P^{*}}{r-\mu}>\frac{M+D'(Q)}{r}$ then the optimal level of the cap, denoted by $\bar{Q}^{*}$, is the root of the following equation:

(18) $$\frac{P^*}{r-\mu}=\frac{M+D'\left(\overline{Q}^*\right)}{r},$$

Proof: Follows from (17) and the convexity of $D(Q)$. □

By *Proposition 3*, if $\frac{P^*}{r-\mu}\le\frac{M+D'(Q)}{r}$, a ban deterring any further market entry is optimal. This is because the expected present value of the flow of market surplus added by the firm entering the market, i.e. $\frac{P^*}{r-\mu}$, does not cover the present value of the flow of social costs, i.e. $\frac{M+D'(Q)}{r}$, associated with the production of one more unit of the good. Otherwise, if $\frac{P^*}{r-\mu}>\frac{M+D'(Q)}{r}$, it is optimal setting a cap at a level higher than the current industry output level Q. Firms will then be allowed to enter the market until the industry output level $\overline{Q}^*$ is reached and where $\frac{P^*}{r-\mu}=\frac{M+D'\left(\overline{Q}^*\right)}{r}$.

Implicit differentiation of (18) yields that:

(19) $$\frac{d\overline{Q}^*}{d\sigma^2}=-\frac{1}{D''\left(\overline{Q}^*\right)}\cdot\frac{M}{(\beta-1)^2}\cdot\frac{d\beta}{d\sigma^2}>0,$$

where the inequality follows from $D''(Q)>0$, $\beta>1$ and $\frac{d\beta}{d\sigma^2}<0$. Thus, the higher the demand uncertainty the larger the optimal cap and the larger the industry output that the regulator is going to allow for. The reason for that is that a higher σ^2 leads, via its effect on the option wedge $\hat{\beta}$, to a higher P^* and, consequently, to a slower entry process in expected terms. This implies that while, on the one hand, the external cost increases at a slower speed, on the other hand, we incur into losses of market surplus since, having a higher entry barrier, P^*, market prices may reach relatively higher levels before a new firm enters the market. Further, one must account for the fact that, even tough, once reached the cap, the external cost stops increasing, there is a loss of market surplus due to the fact that, as no firms may enter the market, the output price evolves freely being absent the barrier control preventing it from going above the level P^*. The loss of market surplus may be relevant and consistently,

(20) $$\lim_{\sigma^2 \to \infty} \overline{Q}^* = \infty,$$

which means that setting an internal cap, $\overline{Q}^*$, is not optimal since restricting firms' entry is too costly in the presence of high levels of market uncertainty.

Implicit differentiation of (18), also yields:

(21) $$\frac{d\overline{Q}^*}{dM} = \frac{1}{D''\left(\overline{Q}^*\right)} \cdot \frac{1}{\beta - 1} > 0,$$

where the inequality follows from $D''(Q) > 0$ and $\beta > 1$. Thus, the higher the production cost the larger the optimal cap and therefore the larger the market size that the regulator is going to allow for. The reason for that is that the larger *M*, the higher the price that triggers entry, i.e. P^*, and, consequently, the slowest the entry process in expected terms. This has, as above, implications for the speed at which the external cost increases and the magnitude of the flow of market surplus.

Last, based on *Proposition 3* and (13), in the case where the optimal cap is at the current *Q*, the expected discounted social welfare is equal to:

(22) $$W^{cap}(X,Q)=\int_0^Q\left[\frac{P(X,q)}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot dq\,,$$

otherwise, when the optimal cap is $\overline{Q}^*$, the expected discounted social welfare is:

(23) $$W^{cap}(X,Q)=\int_Q^{\overline{Q}^*}\left[\frac{P^*}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot\left[\frac{X}{X^*(q)}\right]^{\beta}\cdot dq+$$

$$+\int_0^Q\left[\frac{P(X,q)}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot dq$$

As Dixit and Pindyck (1994, pp. 315-316) show, the term $\left[\frac{X}{X^*(q)}\right]^{\beta}$ is equal to the discount factor $E\left[e^{-r\cdot T(q)}\right]$, where $T(q)$ is the time when process $\{X_t, t>0\}$, starting from its current level X, hits the threshold level $X^*(q)$ for the first time. This insight enables the following, rather intuitive, view of the resulting formula for the welfare function, as captured by (23):

- The last term, $\int_0^Q\left[\frac{P(X,q)}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot dq$, is the integral, over the already supplied Q units of the good, of the expected present value of the flow of social welfare associated with each of those units, i.e. the flow of market surplus, $\frac{P(X,q)}{r-\mu}$, minus the flow of social costs, $\frac{M+D'(q)}{r}$.

- The first term, therefore, represents is the expected present value of the flow of social welfare associated with future entries, those that will add units from the current quantity, Q, to the maximum allowed by the cap, that is, $\bar{Q}^*$. The value of each future entry comprises two parts:

  - $\frac{P^*}{r-\mu} - \frac{M+D'(q)}{r}$, which is the expected present value of the flow of social welfare that the added unit of the good would yield, from the moment in which the firm producing it enters the industry, i.e. when the market price is equal to P^*.
  - $\left[\frac{X}{X^*(q)}\right]^\beta$, the factor by which the payoff $\frac{P^*}{r-\mu} - \frac{M+D'(q)}{r}$ is discounted back to current time.

4.2 Industry equilibrium and welfare under an output tax

Assume that the regulator levies a tax $\tau > 0$ per unit of output.

4.2.1 Optimal entry strategy

The analysis of the industry equilibrium under an output tax is technically identical to the one conducted in Section 3. The only difference is that here the cost for producing one unit of output is equal to $M+\tau$. Hence:

Proposition 4*:* Entry in a perfectly competitive market under an output tax occurs every time the process $\{X_t, t \geq 0\}$ hits the threshold:

(24) $$X^{**}(Q) = \frac{\hat{\beta} \cdot (r-\mu) \cdot \frac{M+\tau}{r}}{\phi(Q)} > X^{*}(Q),$$

or, equivalently, the process $\{P_t, t \geq 0\}$ hits the barrier:

(25) $$P^{**} = \hat{\beta} \cdot (r-\mu) \cdot \frac{M+\tau}{r} > P^{*}$$

Proof: Follows from repeating the proof of *Proposition 1*, this time with a private production cost equal to $M + \tau$. □

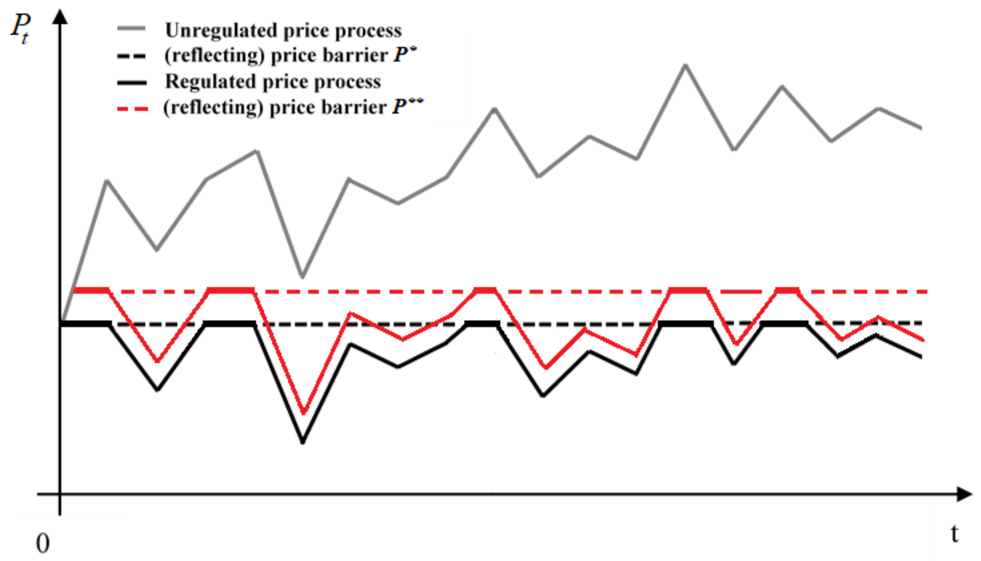

Figure 4: Price dynamics under an output tax

4.2.2 Welfare and the optimal tax rate

The expected discounted social welfare, given the current levels of X and Q, is:

(26) $$W(X,Q,\tau)=C(Q,\tau)\cdot X^{\beta}+\int_0^Q\left[\frac{P(X,q)}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot dq\,.$$

The tax payments from the firms to the government lower their profits and raise the government revenues by the same amount and therefore cancel out of the social welfare. Thus, the only remaining channel by which the output tax affects social welfare is via its effect on the firms' entry thresholds and therefore on entry times. Thus, when setting an optimal tax policy, the regulator is in fact setting an optimal threshold policy. We find that the optimal tax should be set at a level such that both the following *Value Matching Condition*:

(27) $$W_Q\left[X^{**}(Q),Q\right]=0\,,$$

and *Smooth Pasting Condition*:

(28) $$W_{QX}\left[X^{**}(Q),Q\right]=0\,,$$

hold.

As above, Condition (27) is a boundary condition stating that at each market entry we have an increase in welfare associated with the supply of an additional unit of the good, i.e. $\frac{P\left[X^{**}(Q),Q\right]}{r-\mu}-\frac{M+D'(Q)}{r}$, minus the welfare loss associated with the just foregone market entry, i.e. $C_Q(Q,\tau)\cdot X^{**}(Q)^{\beta}$. Condition (28), on the other hand, is an optimality condition that leads to the entry pattern which is optimal from the regulator's perspective and to the tax rate that induces it.

Applying (26) in (27) and (28) leads to the following proposition:

Proposition 5*:* The welfare maximizing tax rate is:

(29) $$\tau(Q) = D'(Q),$$

Substituting (29) in (24) yields that the optimal entry threshold is:

(30) $$X^{**}(Q) = \frac{\hat{\beta} \cdot (r-\mu) \cdot \dfrac{M + D'(Q)}{r}}{\phi(Q)},$$

Notably, by rearranging (30), one may easily show that for any given Q

(31) $$\frac{P^{**}}{r-\mu} = \hat{\beta} \cdot \frac{M + D'(Q)}{r} > \frac{M + D'(Q)}{r}.$$

By (31), in the equilibrium of the considered industry, market entries are always beneficial since the expected present value of the flow of market surplus added by a new firm entering the market, i.e. $\frac{P^{**}}{r-\mu}$, covers always the present value of the flow of social costs, i.e. $\frac{M+D'(Q)}{r}$, associated with the production of one more unit of the good. This is because at each entry the barrier level P^{**} is adjusted upward by taxing at a tax rate τ^{*} which is increasing in Q. Therefore, market entries occur always at a price which is sufficiently high to secure, once accounted for the added external cost, a positive contribution to welfare. Figure 5 provides an illustration of these dynamics.

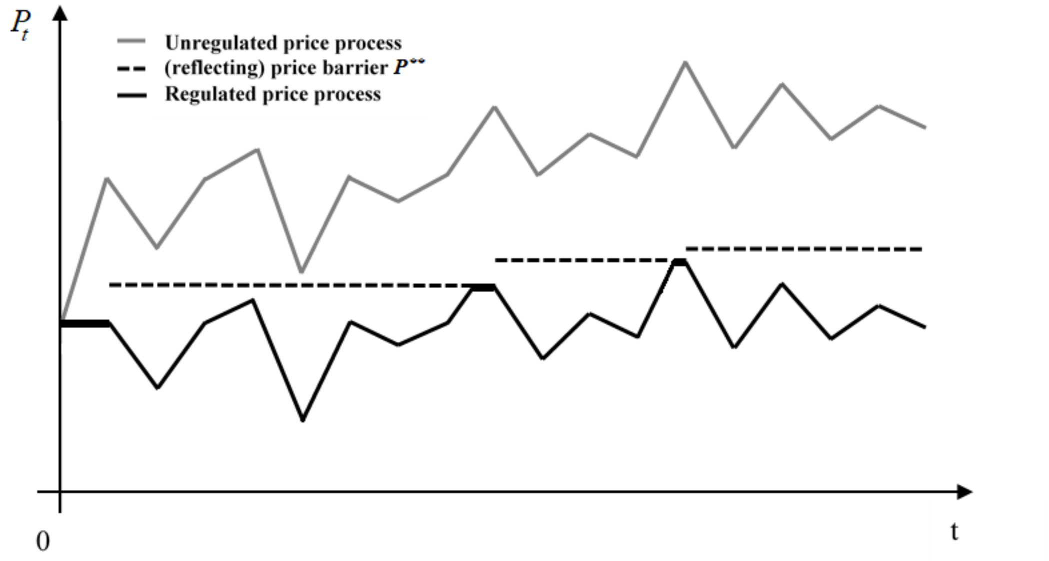

Figure 5: Price dynamics under optimal output taxation

Conditions (27) and (28) also yield:

(32) $$C_Q\left(Q,\tau^*\right) = -\left[\frac{P^{**}}{r-\mu} - \frac{M + D'(Q)}{r}\right] \cdot \frac{1}{X^{**}\left(Q,\tau^*\right)^{\beta}}$$

To integrate (32) we use the following boundary condition:

(33) $$\lim_{Q\to\infty} C(Q,\tau) = 0.$$

The intuition behind Condition (33) is immediate. In (26), the term $C(Q,\tau)\cdot X^{\beta}$ captures the welfare associated with future increases of the industry output. No such changes are expected when $Q \to \infty$ because in that case the entry threshold (24) goes to infinity by the assumption $\lim_{Q \to \infty} \phi(Q) = 0$.

Integrating (32) and applying (33) yields:

(34) $$C\left(Q,\tau^*\right) = \int_Q^\infty \left[\frac{P^{**}}{r-\mu} - \frac{M + D'(q)}{r} \right] \cdot \frac{1}{X^{**}\left(q,\tau^*\right)^\beta} \cdot dq \,.$$

Applying (34) and (1) in (26) yields that the expected discounted social welfare when the tax rate is optimally set is equal to:

(35) $$W^{tax}(X,Q) = \int_Q^\infty \left[\frac{P^{**}}{r-\mu} - \frac{M + D'(q)}{r} \right] \cdot \left[\frac{X}{X^{**}\left(q,\tau^*\right)} \right]^\beta \cdot dq + \int_0^Q \left[\frac{P(X,q)}{r-\mu} - \frac{M + D'(q)}{r} \right] \cdot dq$$

As in (23), the second term, $\int_0^Q \left[\frac{P(X,q)}{r-\mu} - \frac{M+D'(q)}{r} \right] \cdot dq$, is the expected present value of the flow of social welfare associated with the already supplied Q units of the good. The first term represents the expected present value of the flow of social welfare associated with future entries that will add units from the current quantity, Q, up to infinity. The value of each future entry is given by the expected present value of the flow of social welfare that each added unit of the good would yield from the moment in which the firm producing it enters the industry, i.e. $\frac{P^{**}}{r-\mu} - \frac{M+D'(q)}{r}$, discounted back to current time through the stochastic discount factor $\left[\frac{X}{X^{**}\left(q,\tau^{*}\right)}\right]^{\beta}$.

4.3 Social optimum and industry equilibrium under the two policies

Let start by characterizing the social optimum in the considered industry. For the social planner, the relevant problem concerns whether and when to expand the quantity of the good supplied so as to maximize the social welfare (see Dixit and Pindyck, 1994, Chapter 9, Section 1.A).

The expected discounted social welfare, given the current levels of X and Q, is:

(36) $$W(X,Q) = C(Q)\cdot X^{\beta} + \int_{0}^{Q}\left[\frac{P(X,q)}{r-\mu} - \frac{M+D'(q)}{r}\right]\cdot dq$$

Denoting by $X^{SP}(Q)$ the socially optimal threshold for market entry and maximizing (36) subject to:

(37) $W_Q\left[X^{SP}(Q),Q\right]=0$ (*Value Matching Condition*),

(38) $W_{QX}\left[X^{SP}(Q),Q\right]=0$ (*Smooth Pasting Condition*),

(39) $\lim_{Q\to\infty} C(Q)=0$,

yields:

(40) $$X^{SP}(Q) = \frac{\hat{\beta}\cdot(r-\mu)\cdot\frac{M+D'(Q)}{r}}{\phi(Q)} = X^{**}(Q,\tau^{*}) > X^{*}(Q),$$

By (40), it immediately follows that:

Proposition 6*:* A first-best outcome can be achieved through the optimal tax policy while the optimal cap policy may serve only as a second-best alternative.

The optimal tax policy found in the previous section in a decentralized setting leads to the same supply path $\{Q_t, t \geq 0\}$ that the social planner would choose in a centralized setting. This is because in the decentralized case of a regulator choosing an optimal tax policy, the only effect the output tax has on welfare is via the timing of market entries. Formally, the correspondence can be easily proven by substituting $\tau^{*} = D'(Q)$ into Conditions (27) and (28) which would then yield Conditions (37) and (38).

The result above deserves some further comment. For a social planner maximizing welfare, a market entry is desirable as far as the associated gain in terms of market surplus covers its marginal social cost. In our set-up where firms may enter the market at any time point over an infinite time horizon, there is always a time point where this condition is met. Therefore, in a decentralized setting, a first-best policy should be one able to delay market entries so that they occur at the "right" time from the social planner's perspective. Our analysis shows that this is feasible only via price control and, specifically, by equating the tax rate to the marginal external cost associated to the industry output supplied at each time point. This allows a complete internalization of the external cost by the firm when setting the entry strategy and, consequently, the industry equilibrium secures a first-best outcome. In contrast, quantity control exerted through a cap policy may only qualify as a second-best alternative. This is because the resulting industry equilibrium is suboptimal for two reasons. First, the cap has no impact on the timing of market entries since firms keep setting their entry strategy without internalizing the associated external cost and, second, there is a loss of potential welfare gains associated with blocked market entries once the cap has been reached.

From *Proposition 6*, it follows, as a corollary, that

***Proposition** 7:* The welfare achieved through the optimal tax policy, as captured by (35) is higher than the welfare achieved through the optimal cap policy, as captured by (23). The associated welfare gain is equal to:

(41) $$W^{tax}(X,Q)-W^{cap}(X,Q)=$$

$$\int_{\bar{Q}^*}^{\infty}\left[\frac{P^{**}}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot\left[\frac{X}{X^{**}(q,\tau^*)}\right]^{\beta}\cdot dq\ +$$

$$+\int_{Q}^{\bar{Q}^*}\left\{\left[\frac{P^{**}}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot\left[\frac{X}{X^{**}(q,\tau^*)}\right]^{\beta}\ +\right.$$

$$\left.-\left[\frac{P^{*}}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot\left[\frac{X}{X^{*}(q)}\right]^{\beta}\right\}\cdot dq$$

Proof: See Appendix C. □

Eq. (41) illustrates of the sub-optimality of the cap policy. By *Proposition 5*, the first term in (41) is positive for any $Q \leq \overline{Q}^*$. This term represents the welfare that is created only under a tax policy as it springs from units of the good added after the cap level, $\overline{Q}^*$, is reached.. The second integral refers to units produced until $\overline{Q}^*$ is reached and that, therefore, are produced under both policy types. The expression inside the integral shows the welfare trade-off between the two policy tools for units in that range:

- The expected present value of the flow of social welfare is greater under the tax policy since $\frac{P^{**}}{r-\mu} - \frac{M+D'(q)}{r} > \frac{P^*}{r-\mu} - \frac{M+D'(q)}{r}$.
- The expected pace at which these units are added is faster under a cap policy, and therefore under the cap policy the surplus of each unit is expected to be less heavily discounted, as $\left[\frac{X}{X^{**}(q, \tau*)}\right]^{\beta} < \left[\frac{X}{X^*(q)}\right]^{\beta}$.

As shown in Appendix C, the expression inside the second integral is positive as well, implying that for these units the surplus effect dominates the discounting effect.

5. Conclusions

In this paper, we have presented a model of endogenous market structure under uncertainty, with production externalities regulated by a cap on the industry output or via an output tax. The main result is that the tax policy dominates the cap policy when aiming at the maximization of the welfare. In particular, we show that the tax policy allows achieving a first-best outcome since the external cost associated with production is fully internalized.

In the case of a cap policy we have assumed that entry licenses are distributed when the cap is announced. As Bartolini (1995) shows, in the presence of entry licenses, firms holding a license may optimally exercise their option to invest since the threat of being preempted by others is absent. This creates a dynamic entry pattern in which until the cap is reached firms gradually enter the market at time points in which the entry threshold, based on a sufficiently large profitability, is reached. Otherwise, as Bartolini (1993, 1995) shows, if firms are not licensed, this gradual process last only until a certain quantity is reached, and then a competitive run leading the market quantity instantly to the cap is ignited. We do not consider this case because Di Corato and Maoz (2019), have already shown, in a frame similar to ours, that due to the run it yields lower welfare than under rationed entry. Thus, the superiority of the tax policy over the cap policy in the case of licensing implies that it is also better than the cap policy with no licensing.

It should be noticed that the assumption of perfectly competitive firms is crucial for the complete internalization of the external cost. It becomes then of interest, as a potential lead for future research, to extend the analysis in order to study how market power impacts, by distorting the industry output, the degree of internalization, and then to examine whether it also alters the result that the tax policy yields more welfare than the cap policy.

In comparing a tax policy to a cap policy in the presence of externalities, within a centrally planned framework, our study is also related to Weitzman (1974). Via a static model in which the policy maker is facing uncertainty about market outcomes, he concludes that a cap on quantity performs better than a tax if the marginal benefit curve is steeper than the marginal cost curve, and otherwise a tax does better. In contrast, we show the that tax policy is always better than the cap policy regardless of the relation between the two slopes, and for general and standard forms of the demand function and the external cost function. The main reason for the difference between the results springs from the dynamic environment we portray, in contrast to the static analysis by Weitzman. Due to that, while the uncertainty that policy makers face in Weitzman's model is about the current situation, in our model they have a perfect view of current situation and the uncertainty they face is about future development. Thus, at each point in time, the policy makers in our model can fit the best tax rate for the current situation. With a cap policy this is not possible in that, by the very nature of the underlying policy tool, the cap level is assumed to be credibly fixed over a sufficiently long time period. In that sense the dynamic setting gives an advantage to the tax policy.

We have shown that the endogenous entry by firms leads implicitly to a barrier capping the market price. In that sense, the introduction of an output tax is equivalent to a price cap regulation and relates the current study to previous research on the impact of a price cap on irreversible investment under uncertainty in the presence of competition (Dixit, 1991), monopoly (Dobbs, 2004) and oligopoly (Roques and Savva, 2009). The main difference is that while in these papers the price cap is a control used for keeping prices low, in the current study the policy makers adjust this control upwards in order to delay and not foster the market expansion.

Finally, the results of our model are robust to the modification of adding a firm specific production cap alongside the cap on the aggregate industry output. More specifically, assume that the regulator announces a cap $\overline{Q}$ on the aggregate industry output and impose that each firm may produce not more than $0 < \lambda < 1$ units. As one may immediately see, introducing this variation in our model set-up would have no impact on our results. The only thing that one should keep in mind is that in this case i) the number of active firms in the industry is equal to Q/λ and ii) the maximum number of firms entering the market is equal to $\overline{Q}/\lambda$.

APPENDIX

Appendix A – The value of an active firm

In this Appendix, we present the derivation of the value function in (3), i.e. $V(X,Q)$. By a standard no-arbitrage argument (see e.g. Dixit, 1989), $V(X,Q)$ is the solution of the following Bellman equation:

(A.1) $$r \cdot V(Q, X) \cdot dt = \left[P(X,Q) - M\right] \cdot dt + E\left[dV(X,Q)\right],$$

which states that the instantaneous profit, $\left[P(X,Q) - M\right] \cdot dt$, along with the expected instantaneous capital gain, $E\left[dV(X,Q)\right]$, from a change in *X*, must be equal to the instantaneous normal return, $r \cdot V(X,Q) \cdot dt$.

Itô's lemma states that since *X* is a geometric Brownian motion with parameter μ and σ then $V(X,Q)$, being a twice differentiable function of *X* satisfies:

(A.2) $$dV(X,Q) = \left[\tfrac{1}{2} \cdot \sigma^2 \cdot X^2 \cdot V_{XX}(X,Q) + \mu \cdot X \cdot V_X(X,Q)\right] \cdot dt + \sigma \cdot X \cdot dZ .$$

Applying (A.2) in (A.1), taking the expectancy recalling that E*(dZ*) = 0, and rearranging, yields:

(A.3) $$\tfrac{1}{2} \cdot \sigma^2 \cdot X^2 \cdot V_{XX}(X,Q) + \mu \cdot X \cdot V_X(X,Q) - r \cdot V(X,Q) + P(X,Q) - M = 0 .$$

Trying a solution of the type x^b for the homogenous part of (A.3) and a linear form as a particular solution to the entire equation yields:

(A.4) $$V(X,Q) = Z(Q)\cdot X^{\alpha} + Y(Q)\cdot X^{\beta} + \frac{P(X,Q)}{r-\mu} - \frac{M}{r},$$

where $\alpha < 0$ and $\beta > 1$ are the roots of the quadratic equation:

(A.5) $$\tfrac{1}{2}\cdot\sigma^2\cdot x\cdot(x-1) + \mu\cdot x - r = 0.$$

Applying $x = 0$ and then $x = 1$, and bearing in mind that $r > \mu$ asserts that (A.5) has two roots, one of them negative and the other exceeds 1.

The first term in (A.4), namely $\frac{P(X,Q)}{r-\mu} - \frac{M}{r}$, represents the expected present value of the flow of profits conditional on Q remaining forever at its current level. Therefore, the first and second term on the RHS of (A.3) should capture the impact that changes in Q over time have on the value of the firm in expected terms.

By the properties of the Geometric Brownian Motion, when X goes to 0 the probability of ever hitting the barrier triggering a new entry, i.e., $X^*(Q)$, and, consequently, an increase in Q, tends to 0. This leads to the following limit condition:

(A.6) $$\lim_{X\to 0}\left[Z(Q)\cdot X^{\alpha} + Y(Q)\cdot X^{\beta}\right] = 0.$$

Note that as $\alpha < 0$, (A.6) holds only if $Z(Q) = 0$ for any $Q > 0$. Hence, substituting $Z(Q) = 0$ into (A.3) gives (3).

Finally, applying β for x in (A.5) leads to:

(A.6) $$\frac{d\beta}{d\sigma^2} = -\frac{\frac{1}{2}\cdot\beta\cdot(\beta-1)}{\frac{1}{2}\cdot\sigma^2\cdot(2\cdot\beta-1)+\mu} = -\frac{\frac{1}{2}\cdot\beta^2\cdot(\beta-1)}{\frac{1}{2}\cdot\sigma^2\cdot\beta^2+\frac{1}{2}\cdot\sigma^2\cdot\beta\cdot(\beta-1)+\mu\cdot\beta}$$

$$= -\frac{\frac{1}{2}\cdot\beta^2\cdot(\beta-1)}{\frac{1}{2}\cdot\sigma^2\cdot\beta^2+r} < 0,$$

where the third equality follows from (A.4), evaluated at β, and the inequality springs from $\beta > 1$.

Appendix B – Welfare maximization under a cap on the aggregate industry output

Substituting the derivative of (13) with respect to Q in (14), applying (12), and rearranging terms, yields:

(B.1) $$C_Q\left(Q,\bar{Q}\right) = -\left[\frac{P^*}{r-\mu} - \frac{M+D'\left(\bar{Q}\right)}{r}\right]\cdot\frac{1}{X^*\left(Q\right)^\beta},$$

Integrating (B.1) yields:

(B.2) $$C\left(\bar{Q},\bar{Q}\right) - C\left(Q,\bar{Q}\right) = -\int_Q^{\bar{Q}}\left[\frac{P^*}{r-\mu} - \frac{M+D'\left(\bar{Q}\right)}{r}\right]\cdot\frac{1}{X^*\left(Q\right)^\beta}\cdot dq\,.$$

The term $C\left(Q,\bar{Q}\right)\cdot X^\beta$ in (13) captures the welfare associated with future increases of the industry output. No such changes are possible if Q has reached the cap $\bar{Q}$. Therefore, the following boundary condition holds at $Q = \bar{Q}$:

(B.3) $$C(\overline{Q},\overline{Q})=0,$$

Substituting (B.3) in (B.2) yields:

(B.4) $$C\left(Q,\bar{Q}\right)=\int_{Q}^{\bar{Q}}\left[\frac{P^{*}}{r-\mu}-\frac{M+D'\left(\bar{Q}\right)}{r}\right]\cdot\frac{1}{X^{*}\left(Q\right)^{\beta}}\cdot dq\,.$$

Appendix C – Proof of Proposition 6

In this appendix, we show that welfare under an output tax exceeds welfare under a cap on the industry output. The proof is as follows:

(C.1) $$W^{tax}(X,Q)-W^{cap}(X,Q)$$

$$=\int_{\overline{Q}^{*}}^{\infty}\left[\frac{P^{**}}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot\left[\frac{X}{X^{**}(q,\tau^{*})}\right]^{\beta}\cdot dq+$$

$$+\int_{Q}^{\overline{Q}^{*}}\left\{\left[\frac{P^{**}}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot\left[\frac{X}{X^{**}(q,\tau^{*})}\right]^{\beta}-\left[\frac{P^{*}}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot\left[\frac{X}{X^{*}(q)}\right]^{\beta}\right\}\cdot dq$$

$$=\int_{\overline{Q}^{*}}^{\infty}\frac{M+D'(q)}{(\beta-1)\cdot r}\cdot\left[\frac{X}{X^{**}(q,\tau^{*})}\right]^{\beta}\cdot dq+$$

$$+\int_{Q}^{\overline{Q}^{*}}\left\{\frac{M+D'(q)}{(\beta-1)\cdot r}\cdot\left[\frac{X}{X^{**}(q,\tau^{*})}\right]^{\beta}-\frac{M+D'(q)-\beta\cdot D'(q)}{(\beta-1)\cdot r}\cdot\left[\frac{X}{X^{*}(q)}\right]^{\beta}\right\}\cdot dq$$

$$> \int_{Q}^{\bar{Q}^*} \left\{ \frac{M + D'(q)}{(\beta - 1) \cdot r} \cdot \left[\frac{X^*(q)}{X^{**}(q, \tau^*)} \right]^\beta - \frac{M + D'(q) - \beta \cdot D'(q)}{(\beta - 1) \cdot r} \right\} \cdot \left[\frac{X}{X^*(q)} \right]^\beta \cdot dq$$

$$= \int_{Q}^{\bar{Q}^*} \left[M + D'(q) \right] \cdot \frac{\left[h(q) \right]^\beta - 1 + \beta \cdot \left[1 - h(q) \right]}{(\beta - 1) \cdot r} \cdot \left[\frac{X}{X^*(q)} \right]^\beta \cdot dq > 0$$

where:

(C.2) $$h(q) \equiv \frac{X^*(q)}{X^{**}(q, \tau^*)} = \frac{M}{M + D'(q)}.$$

It follows from $D'(q) > 0$ that $0 < h(q) < 1$ for any $q > 0$. This leads to the last inequality in (C.1) which holds because any function of the form $g(x) \equiv x^\beta - 1 + \beta \cdot (1 - x)$ is positive within the range $0 < x < 1$ since:

- $g(0) = \beta - 1 > 0$
- $g(1) = 0$
- $g'(x) = \beta \cdot \left(x^{\beta - 1} - 1 \right) < 0$ for all $0 < x < 1$.

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