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Keywords
Agglomeration, specialization, digital communication, inequality, patents

JEL Codes
J24, O31, O41, R12

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A Tale of Two Cities: Communication, Innovation, and Divergence

Stefano Magrini* Alessandro Spiganti†

Abstract

We present a two-area endogenous growth model where abstract knowledge flows at no cost across space but tacit knowledge arises from the interaction among researchers and is hampered by distance. Digital communication reduces this "cost of distance" and reinforces productive specialization, leading to an increase in the system-wide growth rate but at the cost of more inequality within and across areas. These results are consistent with evidences on the rise in the concentration of innovative activities, income inequality, and skills and income divergence across US urban areas.

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1 Introduction

The idea of a network allowing users to communicate through their personal computers dates back to the 1950s; in 1969, the first message was sent over the Advanced Research Projects Agency Network from a laboratory at the University of California to a second network node at the Stanford Research Institute. Commercial service providers emerged in 1989, marking the beginning of the transition to the modern Internet, whose volume of traffic has doubled approximately every 18 months; its popularity became massive during the 1990s, thanks to the introduction of the World Wide Web and the rise in near-instant communication, through e.g. electronic mail, instant messaging, voice over Internet Protocol telephone calls, and videoconferencing. This changed the way people lived and worked over the last decades, but it is likely that a disproportionally strong impact was felt in all those activities in which knowledge and information sharing are fundamental for production, like research and innovation. Likewise, in the world that will emerge when the pandemic is eventually over, digital communication in general, and videoconferencing in particular, will most probably be integral part of daily working to a much higher extent than before. How does a boost to digital communication change the relative productivity of researchers and their ability to innovate? Which impact does it have on the spatial distribution of these activities and their contribution to growth? What are the repercussions on per capita income and inequality levels?

To investigate these questions, we construct an endogenous growth model with different urban areas and various knowledge spillovers. The economy features two urban areas, each with three sectors: a research sector producing patents using knowledge and skilled labour, an intermediate sector producing differentiated inputs using patents, and a manufacturing sector using skilled labour, unskilled labour, and intermediate inputs. Workers are free to move across areas, and skilled workers can also decide in which sector to work; location and sector decisions are evaluated solely in terms of wage rates. Knowledge takes two forms in the model: abstract and tacit. As in the endogenous growth literature originating from Romer (1986), the former represents codifiable knowledge created during the research effort, which spreads freely throughout the system enhancing the productivity of every researcher. Tacit knowledge is instead all that body of knowledge that cannot be codified, being the non-written heritage of individuals or groups (Polanyi, 1967). This form of knowledge can be transmitted and positively affects the productivity of the researchers, but the flows of tacit knowledge occur essentially through direct, face-to-face, contacts rather than through impersonal means such as patent documents or scientific papers. This introduces a distinction between system-wide and bounded external spillovers on the basis of the type of knowledge being transmitted.

We assume that one urban area is endowed with a more productive research sector, which may parsimoniously reflect a more developed absorptive capacity, i.e. a higher ability to assimilate new knowledge, recognize its value, and apply it to commercial use (Cohen and Levinthal, 1990), or a richer network capital, defined as an area's capacity and capability to access economically beneficial knowledge (Huggins and Thompson, 2014). As a consequence of this productivity gap, geographical specialization arises in equilibrium: the more productive research sector attracts a larger share of researchers and thus the related area specializes in research activities; conversely, the composition of the workforce of the area with a less productive research sector leans towards skilled and unskilled workers producing the final good. Since skilled workers command a higher wage than unskilled ones, the area with a more productive research sector (and thus a relative specialization in research activities) is characterized by higher income per capita; if skilled workers are relatively scarce in the entire population, this area also exhibits a more unequal income distribution. However, the growth rate is the same across areas, since the presence of spillovers means that this only depends on the aggregate flows of new knowledge generated in a period.

We then model a boost to near-instant communication technologies as a fall in the "cost of distance", i.e. a facilitation of the informal interactions among researchers. First, this has a positive effect on the growth rate of the entire economy, since both areas benefit from an increase in the effectiveness of their research effort. Second, a skilled worker becomes relatively more productive if employed in the research sector than in the manufacturing sector, causing a reallocation of skilled workers from manufacturing to research activities. Third, since the more productive research sector is better equipped to exploit these additional interactions (consistently with the interpretation of the productivity of a research sector as its absorptive capacity or network capital), it attracts a larger share of these new researchers, strengthening the previously existing patterns of specialization. As a consequence, this shock increases the previously existing disparities in income per capita and Gini coefficients between areas, as well as the Gini coefficient of the entire system.

Our model is consistent with a series of well-known empirical evidences on innovation and inequality that we recast at the level of metropolitan areas for the United States. In particular, we highlight that, starting from the 1990s, patents have become increasingly spatially concentrated, and there has been increasing divergence in terms of average hourly wages, skills, and patents per capita across metropolitan areas. We also examine the evolution of the cross-sectional distribution of patents per capita using a distribution dynamics approach. We provide evidences supporting the existence of convergence clubs: the polarization identified in the evolution of the unconditional distribution of patents per capita is partly explained by a measure of topography of the area (corrected for natural amenities), which is a commonly used instrument for broadband expansion.

The remainder of this paper is organized as follows. Section 2 reviews previous literature, whereas Section 3 presents some basic facts on patents concentration, inequality, and divergence across metropolitan areas in the United States. Section 4 formalizes the model and Section 5 describes its balanced growth path. Section 6 carries out the comparative statics and presents a numerical example, whereas Section 7 presents some empirical evidences on the evolution over time of the cross-sectional distribution of patents per capita across metropolitan areas. Section 8 concludes.

2 Previous Literature

Our paper is connected to three strands of literature. First, our model is based on the endogenous growth literature originating from Romer (1986), that stresses the role of knowledge as a key driver of productivity and economic growth. In particular, we provide an expanding variety model with knowledge spillovers à la Romer (1990b), where current researchers "stands on the shoulders of past giants". Whereas Romer (1990b) focuses on a single research sector, we modify the model to allow for different areas, so that the growth rate of the entire economy results from the R&D decisions of all areas. In terms of modelling, our paper is similar to models of endogenous technological change with knowledge spillovers across countries, such as Howitt (2000) and Acemoglu et al. (2017). However, differently from these papers, we take a more regional perspective and allow our researchers to move freely across areas and sectors, thus endogenising the spatial distribution of human capital.

Second, we relate to the new economic geography literature, that studies the link between agglomeration and economic integration. Its canonical setting is the so-called core-periphery model (Krugman, 1991), which was embedded in an endogenous growth framework by Baldwin and Forslid (2000) (see Bond-Smith and McCann, 2014, for a literature review). Among the numerous subsequent core-periphery growth models, we share with Bond-Smith and McCann (2020) a focus on innovation, the presence of multiple sectors, and footloose skilled workers (i.e. freely choosing location in response to wage pressure). One of the main differences between our papers concerns the way in which information flows are modelled. They parsimoniously capture knowledge spillovers through exogenous parameters; conversely, we introduce endogenous spillovers based on the endogenous allocation of workers across sectors and areas, thus highlighting the feedback effects among technology, knowledge, agglomeration, and inequality. Related is also a literature that studies urban dynamics using endogenous growth theory, following the seminal contribution of Black and Henderson (1999); its focus is on how local authorities can foster efficient investment in knowledge, but we share an interest on the effect of agglomeration on income inequalities.

Finally, this paper connects to the literature on innovation and agglomeration, which studies how they relate to economic performance and growth (see Carlino and Kerr, 2015, for a literature review). This literature suggests that population and economic activity are spatially concentrated, and that R&D activities are more concentrated than manufacturing activities (e.g. Audretsch and Feldman, 1996, Buzard et al., 2017). One of the underlying explanation, which dates back to Marshall (1890), is that geographic proximity facilitates the transfer of knowledge, especially through serendipitous interactions among workers and firms. However, there is a growing base of evidence suggesting that knowledge is increasingly being shared across geographic clusters, but through more selective routes that require conscious investments, absorptive capacity, and network capital (see Huggins and Thompson, 2014, for a review). In this paper, we take as given that one area is endowed with a research sector relatively more effective at exploiting the knowledge spillovers and analyse theoretically the resulting spatial allocation of innovative activities.

3 Some Empirical Facts

We report some empirical facts regarding innovation and inequality in the United States in the last decades. Our unit of geography is the metropolitan statistical area (MSA) i.e. "a region consisting of a large urban core together with surrounding communities that have a high degree of economic and social integration with the urban core" (Ruggles et al., 2020). We pick MSAs as our geographic entities for various reasons. First, MSAs represent economic spatial units and so are considered more appropriate to study income convergence than states, regions, or even counties (e.g. Drennan, 2005); moreover, they are more consistent with our theoretical model. Second, innovation is mainly an urban phenomenon.[1] Third, there is large heterogeneity across MSAs in terms of wages, wage disparities, and capacity to innovate.

We measure innovation with the number of patents granted by the USPTO.[2] We locate each patent according to the US location in which

[1] For example, the vast majority of innovators in our dataset from the United States Patents and Trademarks Office (USPTO) reside in a metropolitan area (approximately 95%).

[2] A patent is an exclusionary right conferred for a set period to the patent holder, in exchange for sharing the details of the invention. As common in the literature, we restrict our attention to utility patents, which cover the creation of a new or improved product, process or machine; these approximately cover 90% of all patents (they exclude design patents and plant patents).

the inventor of the innovation resides, which is extracted from patent text and used to determine latitude and longitude; then, we assign that location to its current MSA.[3] When a patent is coauthored by more than one inventor, we split it equally among them.

To provide some anecdotal evidence on inequality, we draw data from the Census Integrated Public Use Micro Samples (IPUMS, Ruggles et al., 2020), which reports for each decade between 1950 and 2010 individual-level information on demographic and socio-economic indicators, including data on wage received and education; moreover, it also provides the metropolitan area of residence of each individual.

3.1 Innovation

It is well known that the number of patents issued by the USPTO annually has steadily increased since the 1990s, as shown for example in Figure 1a. But what about the spatial distribution of these innovating activities? In general, R&D activities are more concentrated than manufacturing activities (e.g. Buzard et al., 2017); moreover, both Andrews and Whalley (2021) and Forman and Goldfarb (2021) report a particularly pronounced increase in the geographic concentration of patenting at the US county level starting from the 1990s.

Following Andrews and Whalley (2021), we measure concentration using Ellison and Glaeser's (1997) "dartboard approach". This consists in calculating an index of the spatial concentration of innovation intensity by comparing the observed spatial distribution of patents to what it would have been if it was proportional to population distribution.[4] In particular, for each year t and all metropolitan statistical areas $n \in N$, our dartboard

[3] We use the 2019 Cartographic Boundary Files provided by the United States Census Bureau, `https://www.census.gov/geographies/mapping-files/time-series/geo/cartographic-boundary.html`. The analysis is run using Stata 16 by StataCorp (2019).

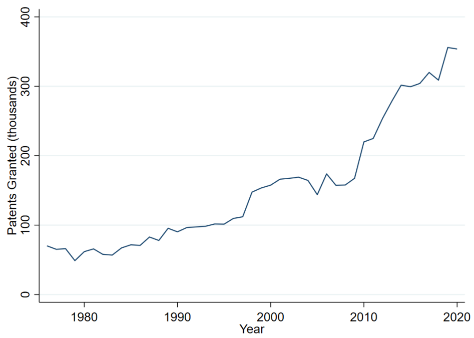
[4] Population data at the county level are provided by Manson et al. (2020); we linearly interpolate population for years between census years (only for years before 2010; afterwards data are provided for each year) and then aggregate counties to their current MSA. We use definitions provided by the National Bureau of Economic Research, `https://www.nber.org/research/data/census-core-based-statistical-area-cbsa-federal-information-processing-series-fips-county-crosswalk`.

innovation intensity concentration index is

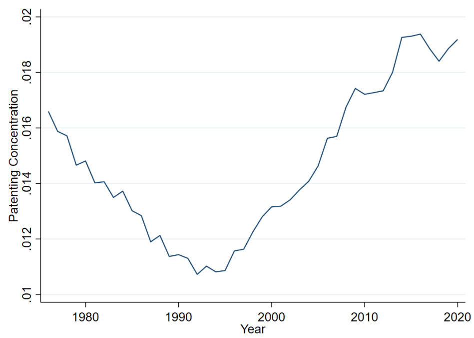
$$Concentration_t = \frac{\sum_{n=1}^{N} \left(SharePat_{nt} - SharePop_{nt}\right)^2}{1 - \sum_{n=1}^{N} SharePop_{nt}^2}, \tag{1}$$

where $SharePat_{nt}$ and $SharePop_{nt}$ are, respectively, the shares of patents granted and of population living in area n in year t. The scale of this index is such that a value of zero can be interpreted as indicating a complete lack of agglomerative forces, whereas a value of one would indicate that all patenting occurs in one geographic area.

The evolution of this index is reported in Figure 1b, which shows a decline in concentration across metropolitan statistical areas between 1976 and the beginning of the 1990s, followed by a sharp increase in patenting concentration.



(a) Number of Patents



(b) Patenting Concentration

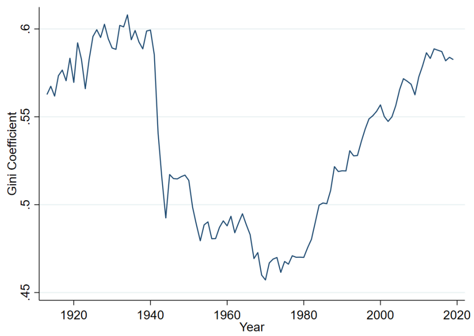
Figure 1: Innovation in the United States

Notes. The first panel shows the number of utility patents granted by USPTO in any given year between 1976 and 2020. The second panel shows the dartboard innovation intensity concentration index across metropolitan statistical areas in the United States between 1976 and 2020. Own elaborations using data from USPTO.

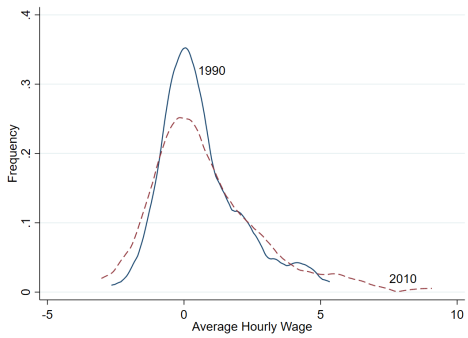
3.2 Inequality

As documented by e.g. Piketty and Saez (2003) and Atkinson et al. (2011), following several decades of wage compression and increasing equality, starting around 1980 income inequality has risen sharply in the United States. This trend, which has been named "great divergence" by Paul Krugman, is evident in Figure 2a, which shows the evolution of the US

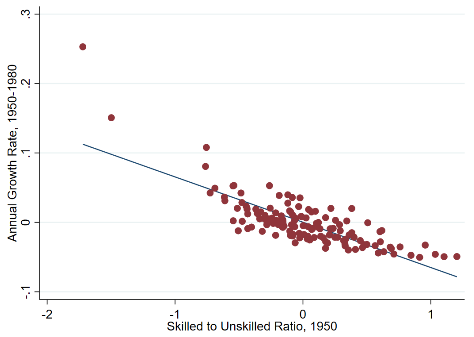
Gini coefficient over the last hundred years. However, the same term has been used by Moretti (2012) to describe a different process, whereby there has been, approximately from the same time, increasing divergence between leading cities and poorer cities (see also Berry and Glaeser, 2005, Giannone, 2021). Figure 2b provides a first look at this divergence across MSAs: by plotting the distributions of the average hourly wages across the set of MSAs in 1990 and 2010, it shows a spreading out over this time period.



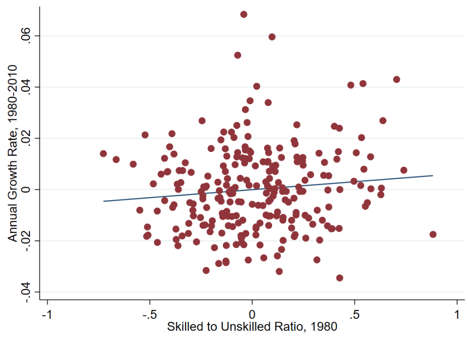
(a) Gini Coefficient



(b) Distribution of MSAs



(c) Skill Convergence, 1950-1980



(d) Skill Divergence, 1980-2010

Figure 2: Divergence in the United States

Notes. The first panel reports the Gini coefficient calculated using pre-tax national income (from labour and capital) of US adults between 1913-2019 (source: World Inequality Database). The second panel provides kernel estimations of the distributions of (a balanced panel of) MSAs according to the (demeaned) average hourly wages (at constant 1999 prices) in 1990 and 2010 (note that data may be censored, with amounts higher than a certain time-changing top code value expressed as the state medians of values above it). The third and fourth panels show each MSA's annual average growth (demeaned) against its (demeaned and in natural logarithm) initial level of the ratio between highly educated (at least 4 years of college) and less educated (everyone else) respondents; lines are linear fit. Own elaboration using data from IPUMS (Ruggles et al., 2020).

Moretti (2012) argues that one reason for the increasing divergence across cities is due to a divergence of skills. Using education as a proxy for skills (like e.g. Acemoglu and Autor, 2011), Figure 2c and 2d show the relationship between the growth rate in the ratio between the number of highly and less educated workers living in a MSA and the value of this ratio in the initial period, respectively for 1950-1980 and 1980-2010. As already observed by Giannone (2021), before 1980 there was convergence in the skill ratio across MSAs, whereas afterwards skills diverged over space (as reported by e.g. Moretti, 2004).

4 The Model

We consider an infinite-horizon economy in continuous time. This is inhabited by a continuum of infinitely-lived agents comprising a constant mass H of skilled workers and a constant mass L of unskilled workers. The economy features two urban areas, i and j. Each area has three sectors: a research sector which produces patents using knowledge and skilled labour, an intermediate sector producing differentiated intermediate inputs using forgone final good and patents, and a manufacturing sector producing a homogeneous good using skilled labour, unskilled labour, and intermediate inputs. Unskilled workers are employed in the manufacturing sector and are free to move across areas; a skilled worker is employed in either the research sector or the manufacturing sector, and can freely move across areas and sectors. Locations and sectors are evaluated solely in terms of wage rates.

4.1 The Agents

Agents, indexed by z, are infinitely-lived and have an instantaneous constant relative risk aversion utility function, meaning that they each maximize, subject to a budget constraint,

$$\int_{t=0}^{\infty} e^{-\rho t} \frac{c_z(t)^{1-\sigma} - 1}{1-\sigma} dt, \tag{2}$$

where $c_z(t)$ is the consumption of agent z at time t, $\rho > 0$ is the subjective discount rate, and $1/\sigma > 0$ measures the willingness to substitute intertemporally. Agents inelastically supply one unit of labour and own equal shares of all the area's firms.

Agents consume a unique final good that can be transported between the two areas at no cost; therefore, all consumption arising from the system can be aggregated in the system-wide variable $C(t)$. The maximization problem of the agents results in the usual consumption Euler's equation, which relates the interest rate $r(t)$ to the rate of growth of consumption according to

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\sigma}. \tag{3}$$

Here, we concentrate on the case in which the growth rate of consumption is positive, which implies $r(t) > \rho$. To ensure that the integral in (2) converges, the rate of growth of current utility is assumed to be smaller than the rate of time preference, i.e.

Assumption 1. $(1 - \sigma)\, \dot{C}(t)/C(t) < \rho$.

4.2 The Manufacturing Sector

The final good is produced competitively by a representative firm using unskilled labour, skilled labour, and a set of intermediate inputs. The available variety of intermediate inputs in a urban area at any point in time is taken as given by the firm and consists of the sum of inputs produced in the same area and inputs imported from the other area (as in e.g. Rivera-Batiz and Romer, 1991, Rivera-Batiz and Xie, 1993). The intermediate inputs depreciate fully after use.[5] Below and in the next subsections, we describe i's sectors, but the same applies to j's; for ease of reading, we drop the time index.

Define A_i and A_j as the number of intermediate inputs designed and produced in i and j, respectively. Let the quantity of any intermediate input produced in i and employed in the same urban area be $x_i(a_i)$, with

[5] This is a standard assumption in the expanding variety models. Indeed, it simplifies the exposition considerably, since the past amounts of these inputs are not additional state variables. However, results without this assumption are identical.

$a_i \in A_i$; analogously, the quantity of any intermediate input produced in j and employed in i is $x_i(a_j)$, with $a_j \in A_j$. The overall production structure in i's final sector is represented by the following additively separable function:

$$M_i = L_i^{\alpha} H_{m,i}^{\beta} \left[\int_0^{A_i} x_i(a_i)^{\gamma} da + \int_0^{A_j} x_i(a_j)^{\gamma} da \right] S_{m,i}, \tag{4}$$

where M_i is the final good produced in i, $H_{m,i}$ represents skilled labour employed in i's manufacturing sector, and $S_{m,i}$ reflects the size of spillovers arising from the interaction between skilled workers employed within the same urban area.[6] Formally, these intra-area spillovers are parametrized through the following equation,

$$S_{m,i} = H_{r,i}^{\phi_r} H_{m,i}^{\phi_m}, \tag{5}$$

where $H_{r,i}$ represents skilled labour employed in i's research sector, while ϕ_r and ϕ_m measure spillover elasticities.[7] In the remaining of this paper, we focus on the more interesting case by taking the following assumption:

Assumption 2. $(\phi_m + \phi_r)/\phi_m > 0.$

The Cobb-Douglas formulation of the production function in (4) leads to iso-elastic demand curves; in particular, the demands of intermediate inputs by the final good producer in area i are

$$x_i(a_i) = \gamma^{\frac{1}{1-\gamma}} L_i^{\frac{\alpha}{1-\gamma}} H_{m,i}^{\frac{\beta}{1-\gamma}} S_{m,i}^{\frac{1}{1-\gamma}} p_i(a_i)^{-\frac{1}{1-\gamma}} \tag{6a}$$

$$x_i(a_j) = \gamma^{\frac{1}{1-\gamma}} L_i^{\frac{\alpha}{1-\gamma}} H_{m,i}^{\frac{\beta}{1-\gamma}} S_{m,i}^{\frac{1}{1-\gamma}} p_i(a_j)^{-\frac{1}{1-\gamma}}, \tag{6b}$$

where $p_i(a_i)$ and $p_i(a_j)$ are the prices of an intermediate good sold in i but produced in i and j, respectively. Implicitly, we are assuming the absence of transportation costs for intermediate goods across areas.

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[6] This type of local spillovers have a long-tradition in economics, see e.g. Jacobs (1970).

[7] The main findings of the model are qualitative the same, but more analytically complicated, if the externality effect depends positively on the average level of human capital (similarly to e.g. Lucas, 1988, Black and Henderson, 1999, Moretti, 2004); see Appendix A.2.

The final sector operates in a perfectly competitive setting, hence $\alpha + \beta + \gamma = 1$. To ensure that the wage rate earned by skilled workers is higher than the wage rate earned by unskilled workers, we assume

Assumption 3. $H_{m,i}/L_i < \beta/\alpha$.

For simplicity, we assume that the final good is traded freely within the system in the absence of any transportation cost. As a consequence, in equilibrium its price must be the same in both urban areas, and we normalize it to one.

4.3 The Research Sector

Following the large literature originated from Romer (1990a,b), the research sector produces knowledge in the form of designs for new intermediate inputs, using skilled labour and existing knowledge. Formally, the flow of new knowledge, i.e. the number of new designs, created in urban area i at any point in time is given by:

$$\dot{A}_i = \delta_i H_{r,i}^{\eta} S_{r,ij} A, \tag{7}$$

where $H_{r,i}$ is the number of *researchers* in i, $0 \leq \eta < 1$ is a parameter inducing decreasing returns in its stock (similarly to Kortum, 1993, Jones, 1995), $\delta_i > 0$ is an exogenous parameter characterizing the productivity of the local research system, A is an index of the economy technology frontier (which will be endogenized below), and $S_{r,ij}$ reflects inter-area spillovers in research. This form of the innovation possibility frontier implies that new knowledge in i results from the effort of the researchers in the area, but the effectiveness of these efforts more generally depends on the research done in the entire economy.

Indeed, equation (7) introduces two types of spillovers. First, there is a positive a-spatial spillover coming through the economy technology frontier, A. This is assumed to be given by

$$A \equiv A_i + A_j, \tag{8}$$

meaning that A simply represents the aggregate number of designs already existing, or, equivalently, the overall level of abstract knowledge created so far and available to all researchers.[8] Second, there is a positive network effect between the researchers in the two areas; this represents the flow of tacit knowledge, which occurs essentially through informal interactions and exchange of ideas. Similarly to the intra-area ones, we assume that these inter-area spillovers have the following representation:

$$S_{r,ij} = H_{r,i}^{\psi_1} H_{r,j}^{\psi_2} \nu_i^{\psi}, \tag{9}$$

with the function ν_i expressing the effectiveness of the interaction to the benefit of i and the parameters ψ_1, ψ_2, and ψ governing the strength of the impact of each component (i.e. researchers in the area, researchers in the other area, and the effectiveness, respectively). We take the following assumption:

Assumption 4. $1 - \eta > \psi_1 - \psi_2$.

This assumption eases calculations since, as clarified in Appendix A.1, it is a sufficient condition for the stability of the equilibrium allocation of researchers across urban areas.

It is well-known that any sort of distance, d, between the researchers of the two areas, being geographical or technological, may make these informal interactions more difficult (see e.g. Jaffe et al., 1993); however, a natural assumption is that a higher productivity of the local research system, which may partly be intended as its absorptive capacity (Cohen and Levinthal, 1990) or its network capital (Huggins and Thompson, 2014), may not only facilitate the exploitation of these interactions but also (partly) compensate for the distance. As a consequence, we let $\nu_i \equiv \nu(\delta_i, d)$ and $\nu_j \equiv \nu(\delta_j, d)$ and we take the following assumption:

Assumption 5. *The function $\nu(\delta, d)$ is twice differentiable in δ and d, and satisfies*

$$\frac{\partial \nu(\delta, d)}{\partial \delta} \geq 0, \quad \frac{\partial \nu(\delta, d)}{\partial d} \leq 0, \quad \frac{\partial^2 \nu(\delta, d)}{\partial d \partial \delta} \leq 0, \quad \frac{\partial}{\partial \delta} \left| \frac{d}{\nu(\delta, d)} \frac{\partial \nu(\delta, d)}{\partial d} \right| > 0,$$

[8] The qualitative results are unaffected as long as the economy technology frontier is a linearly homogeneous function of the number of intermediate inputs in the two areas, e.g. equal to the technology level of the most advanced area or an average of the two.

where the last condition ensures that the d-elasticity of $\nu(\delta, d)$ increases with δ.

4.4 The Intermediate Sector

The intermediate sector in area i is composed of an infinite number of firms on the interval $[0, A_i]$. Each of these firms has purchased a patent from the research sector and can then produce the related intermediate input at marginal cost equal to $\kappa > 0$ units of the final good. We assume that this marginal cost is strictly higher if the intermediate input is manufactured in the other area, thus excluding the existence of an inter-area trade of patents.

In line with the endogenous technological change literature, an intermediate producer acts as a monopolist in the production of its particular intermediate input. An intermediate firm in i faces the demand $x_i(a_i)$ in (6a) from the final producer in i with the corresponding price $p_i(a_i)$ and the demand $x_j(a_i)$ at price $p_j(a_i)$ from the final producer in j; let aggregate demand faced by an intermediate firm in i be $X(a_i) \equiv x_i(a_i) + x_j(a_i)$. Since demands are iso-elastic, the monopoly price is a constant mark-up over marginal cost. Without loss of generality, we normalise the marginal cost of machine production to $\kappa \equiv \gamma$, so that

$$p \equiv p_i(a_i) = p_j(a_i) = \kappa\gamma^{-1} = 1. \tag{10}$$

Intermediate inputs all have the same price across intermediate firms and areas, since the marginal cost is the same. Intermediate inputs depreciate fully after use, and so p can also be interpreted as a rental price or the user cost of the input.

Substituting (10) into (6) shows that a manufacturing firm demands the same quantity of each intermediate input, irrespective of their origin,

$$x_i = \gamma^{\frac{1}{1-\gamma}} L_i^{\frac{\alpha}{1-\gamma}} H_{m,i}^{\frac{\beta}{1-\gamma}} S_{m,i}^{\frac{1}{1-\gamma}}. \tag{11}$$

As a consequence, the intermediate input producers located in the two different areas all face the same aggregate demand, $X = x_i + x_j$, and enjoy the same instant profits, $\pi = X(1-\gamma)$. Hence, final good production simplifies to

$$M_i = AL_i^{\alpha} H_{m,i}^{\beta} x_i^{\gamma} S_{m,i}. \tag{12}$$

The decision about undertaking the production of a new intermediate input is taken comparing the discounted value of the flow of future profits to the cost of the initial investment in acquiring a patent from the research sector. With this knowledge, the monopolistically competitive research sector sets the price of a patent equal to the present value of the stream of future profits of the intermediate sector's monopolist. Therefore, the cost of a patent, irrespective of its location, is $P = \int_{t=0}^{\infty} \pi(t) e^{-rt} dt$. Patents are infinitely lived; hence, if the interest rate is constant,

$$P = \frac{X(1-\gamma)}{r}. \tag{13}$$

5 The Equilibrium

We now characterize the equilibrium; when necessary to avoid confusion, we reintroduce time indexes. An allocation is defined by time paths of consumption levels $[C(t)]_{t=0}^{\infty}$, aggregate spending on intermediate inputs $[X_i(t), X_j(t)]_{t=0}^{\infty}$, labour allocations $[H_{m,i}(t), H_{m,j}(t), H_{r,i}(t), H_{r,j}(t), L_i(t), L_j(t)]_{t=0}^{\infty}$, available intermediate input varieties $[A_i(t), A_j(t)]_{t=0}^{\infty}$, and time paths of interest rates $[r(t)]_{t=0}^{\infty}$, wage rates in the research sectors $[w_{r,i}(t), w_{r,j}(t)]_{t=0}^{\infty}$, wage rates for skilled and unskilled workers in the manufacturing sectors $[w_{m,i}(t), w_{m,j}(t), w_{l,i}(t), w_{l,j}(t)]_{t=0}^{\infty}$, quantities of each intermediate input $[x_i(t), x_j(t)]_{t=0}^{\infty}$, and patent costs $[P(t)]_{t=0}^{\infty}$. An equilibrium is an allocation in which final good producers, research firms, and intermediate good producers choose, respectively, $[H_{m,i}(t), H_{m,j}(t), L_i(t), L_j(t), x_i(t), x_j(t)]_{t=0}^{\infty}$, $[H_{r,i}(t), H_{r,j}(t), P(t)]_{t=0}^{\infty}$, and $[x_i(t), x_j(t)]_{t=0}^{\infty}$ as to maximize (the discounted value of) profits, the evolution of wages and interest rate is consistent with market clearing, agents make labour and consumption decisions as to maximize their lifetime utility, and the evolution of $[A_i(t), A_j(t)]_{t=0}^{\infty}$ is determined by free entry.

In particular, we focus on a balanced growth path, i.e. an equilibrium in which aggregate variables, like consumption $C(t)$ and output $M(t)$, grow at the same constant rate as system-wide abstract knowledge, $g \equiv \dot{A}(t)/A(t)$

for all t. This is possible, from equation (3), only if the interest rate is constant: we thus look for an equilibrium in which $r(t) = r$ for all t.

Assuming for a moment that the labour market is characterized by a stable allocation of both unskilled and skilled labour across areas and sectors, then it is clear from equations (11) that the equilibrium demands of intermediate inputs would also be constant, $x_i(t) = x_i$ and $x_j(t) = x_j$ for all t; as implied by (13), in such an equilibrium, also the price of a patent is constant over time, $P(t) = P$ for all t. Under such a constant allocation of resources, equation (12) ensures that the output in both urban areas, $M_i(t)$ and $M_j(t)$, grows at the same rate as system-wide abstract knowledge, g. As a consequence, aggregate output, $M(t)$, also grows at g. Therefore, in an economy characterized by a constant allocation of unskilled and skilled labour across areas and sectors, a balanced growth path allocation exists in which

$$\frac{\dot{M}(t)}{M(t)} = \frac{\dot{M}_i(t)}{M_i(t)} = \frac{\dot{M}_j(t)}{M_j(t)} = \frac{\dot{C}(t)}{C(t)} = \frac{\dot{A}(t)}{A(t)} \equiv g.$$

To solve the model for this balanced growth equilibrium it is therefore necessary to determine the equilibrium allocation of workers across areas and sectors, and to verify that this allocation is consistent with a constant interest rate.

5.1 The Equilibrium Allocation of Workers

In this section, we characterize the allocation of skilled and unskilled workers across areas and sectors.

5.1.1 The Inter-Area Allocation of Researchers

We first take the aggregate number of researchers, $H_r \equiv H_{r,i} + H_{r,j}$, as given; this will be endogenized below. From the maximization problem of a firm in the research sector, the wage rate for a researcher in urban area i, $w_{r,i}$, must satisfy the first order condition $w_{r,i} = \partial(P\dot{A}_i)/\partial H_{r,i}$. Using equations (7) and (13), this wage rate is

$$w_{r,i} = AX\eta\delta_i H_{r,i}^{\eta-1} S_{r,ij} \frac{1-\gamma}{r}. \tag{14}$$

Any skilled worker is free to enter either research sector: in equilibrium, researchers must receive the same compensation across the two areas, i.e. $w_{r,i} = w_{r,j} \equiv w_r$. The following equilibrium allocation ensues:

$$\frac{H_{r,i}}{H_{r,j}} = \left(\frac{\delta_i}{\delta_j}\right)^{\frac{1}{1-\eta-\psi_1+\psi_2}} \left(\frac{\nu_i}{\nu_j}\right)^{\frac{\psi}{1-\eta-\psi_1+\psi_2}}. \tag{15}$$

For given distance and research productivities, the equilibrium spatial allocation of skilled labour in research is thus constant. Moreover, by Assumptions 4 and 5, there is a positive relationship between productivity in research and the relative concentration of research activities: an urban area characterized by a relatively higher productivity of the research sector will attract a larger share of researchers.

5.1.2 The Allocation of Workers in the Manufacturing Sector

The manufacturing sectors are competitive, hence the wage rate of unskilled and skilled workers employed in area i are, respectively

$$w_{l,i} = \frac{\partial M_i}{\partial L_i} = \alpha L_i^{\alpha-1} H_{m,i}^{\beta} A x_i^{\gamma} S_{m,i} \tag{16a}$$

$$w_{m,i} = \frac{\partial M_i}{\partial H_{m,i}} = \beta L_i^{\alpha} H_{m,i}^{\beta-1} A x_i^{\gamma} S_{m,i}. \tag{16b}$$

Since workers can freely move between the two manufacturing sectors, in equilibrium unskilled and skilled workers must receive the same compensation across areas, i.e. $w_{l,i} = w_{l,j} \equiv w_l$ and $w_{m,i} = w_{m,j} \equiv w_m$. This implies $L_i/L_j = H_{m,i}/H_{m,j}$; consequently, $H_{m,i}/L_i = H_{m,j}/L_j = H_m/L$, where $H_m \equiv H_{m,i} + H_{m,j}$ is the aggregate number of skilled workers employed in the manufacturing sector. Moreover, by combining this result with (11), we prove in Appendix A.1 that $L_i/L_j = x_i/x_j$ and that the external effects are endogenously equalized, $S_{m,j} = S_{m,i}$. In turn, this implies that, in equilibrium,

$$\left(\frac{H_{r,j}}{H_{r,i}}\right)^{\frac{\phi_r}{\phi_m}} = \frac{H_{m,i}}{H_{m,j}} = \frac{L_i}{L_j} = \frac{x_i}{x_j}. \tag{17}$$

These ratios are constant along the balanced growth path given (15).

5.1.3 The Inter-Sector Allocation of Skilled Workers

Finally, the intra-area equilibrium requires inter-sectoral wage equalisation for skilled workers, $w_{m,i} = w_{r,i}$ and $w_{m,j} = w_{r,j}$. Given the inter-area equilibrium allocation of researchers, these conditions become $w_m = w_r \equiv w_h$, where w_h is the unique wage paid to a skilled worker across sectors and areas. We show in Appendix A.1 that this condition is met when

$$\frac{H_r}{H_m} = \frac{\eta(1-\gamma)\gamma}{\beta}\left(\frac{r-\rho}{r\sigma}\right). \tag{18}$$

Condition (18) maintains that the equilibrium allocation of the given stock of skilled labour depends on parameters and the endogenous interest rate. Since the interest rate must be constant along the balanced growth path, the proportional allocation of skilled workers in the research sector and in the final good sector also remains constant.

5.2 The Equilibrium Growth Rate

We showed in Section 5.1 that the system is characterized by a constant allocation of workers across sectors and urban areas. Given that such a constant allocation exists, the economy exhibits a balanced growth path. To complete the characterization of the balanced growth path, note that free entry into research implies

$$\eta\delta_i S_{r,ij} A H_{r,i}^{\eta-1}\frac{X(1-\gamma)}{r} = w_h, \tag{19}$$

where the left hand side is the private return from hiring one more researcher and the right hand side is the related flow cost. Together with (16b), this implies that the equilibrium interest rate must be $r = \eta(1-\gamma)\delta_i\gamma\beta^{-1}S_{r,ij}H_{r,i}^{\eta-1}H_m$, which is constant under the constant allocation of workers.

Proposition 1. *The system exhibits a globally stable balanced growth path equilibrium in which output, consumption, physical capital, aggregate abstract knowledge, abstract knowledge in each area, and wages grow at the* *same constant rate given by*

$$g = \delta_j S_{r,ji} H_{r,j}^{\eta-1} H_r = \delta_i S_{r,ij} H_{r,i}^{\eta-1} H_r. \tag{20}$$

Along the balanced growth path, the price of a patent, the price of each intermediate input, the price of the final good, the interest rate, and the labour allocations across sectors and areas are constant.

Proof. See Appendix A.1 □

5.3 Income, Inequality, and Growth in Urban Areas

In this section, we evaluate whether differences in income per capita levels and growth rates arise between the two urban areas along the balanced growth path. Without loss of generality, we assume that area i is endowed with a more productive research sector, i.e.

Assumption 6. $\delta_i > \delta_j$.

Our first result characterizes the relative specialization of skilled labour between the two areas.

Proposition 2. *Along the balanced growth path, the urban area with a relatively more productive research sector is characterized by a relative specialization in research activities, i.e.* $H_{r,i}/H_{r,j} > H_{m,i}/H_{m,j} = L_i/L_j$.

Proof. Condition (15), Assumption 5, and Assumption 6 imply $H_{r,i} > H_{r,j}$; the rest follows from condition (17). □

Having established the relative productive specialization of the urban areas, we can turn our attention to disparities in income levels. The level of income in each urban area, its GDP, is calculated as the summation of the wages of its workers, since profits are driven down to zero by competition or free entry. Thus, the overall GDP level in i can be expressed as

$$Y_i = w_l L_i + w_h H_{m,i} + w_h H_{r,i}. \tag{21a}$$

Corollary 2.1. *Along the balanced growth path, a relative specialization in research activities is a sufficient condition for a constantly higher level of GDP per worker.*

Proof. See Appendix A.1 □

In area i, there are $H_{r,i}+H_{m,i}$ skilled workers earning w_h and L_i unskilled workers earning w_l. With two income levels, the Gini coefficient, G_i, is simply the difference between the proportion of all income accruing to the high income group and the proportion of agents in the high income group, i.e.

$$G_i = \frac{(H_{r,i} + H_{m,i})\, w_h}{Y_i} - \frac{H_{r,i} + H_{m,i}}{L_i + H_{r,i} + H_{m,i}}. \tag{22}$$

Corollary 2.2. *If skilled workers are sufficiently scarce, a relative specialization in research activities is a sufficient condition for a constantly higher Gini coefficient.*

Proof. See Appendix A.1 □

Finally, we consider the effect of the relative specialization of the urban areas on the growth rates of their income levels.

Corollary 2.3. *Along the balanced growth path, GDP per worker grows in both urban areas at the constant rate g, irrespective of the areas' specialization.*

Proof. Along the balanced growth path, wages grow at g whereas labour allocations are constant. Thus, the areas' GDP levels in (21) must also grow at rate g. Since labour allocations are constant, GDP per worker also grows at g in both areas. □

Therefore, the urban area whose research system is more productive features a relative specialization in research activities compared to the other urban area and enjoys a permanently higher level of GDP per worker but, possibly, a more unequal society. However, the growth rates are the same.

Several empirical papers report a positive relationship between metro size and inequality (e.g. Baum-Snow and Pavan, 2013, Florida and Mellander, 2016). This empirical prediction is matched by our theoretical model if we further impose $\phi_r/\phi_m < 0$. This is the case if, for example, intra-area spillovers positively depends on the proportion of skilled workers employed in research in an area; that urban productivity may relate to the density of researchers is well-established in the literature (e.g. Abel et al., 2012).

6 Communication, Productivity, and Specialization

In this section, we focus on a positive technology shock, intended as a boost to near-instant communication technologies, that in terms of the model translates into a reduction in the distance involved in inter-urban relations among researchers, d. Appendix A.3 presents results following a relative change in the productivity of research, that could be either specific to an area, e.g. on the parameters δ_i or δ_j, or common to the system, e.g. on ψ.

6.1 A Positive Technology Shock

A firm in the research sector needs knowledge and information, in addition to labour: the flow of tacit knowledge, which occurs through informal interactions and exchange of ideas, not only allows to keep up with scientific and technological advancements, but also to gain timely access to problems, needs, and requests that may direct its activity. In this regard, the diffusion of near-instant communication technologies and videoconferencing certainly plays an important role. Their importance, however, is likely to depend on the features of the network of relations in which they are employed: their effectiveness is probably stronger when these tools are adopted within an already established network (Cohen and Levinthal, 1990, Huggins and Thompson, 2014).[9]

Consistently with this interpretation, we assume that it is within the inter-area networks of relations that these tools are more likely to be suc-

[9] The positive effect of the diffusion of communication services on growth is well documented in the literature, at least since Hardy (1980); see Kolko (2012) and Castaldo et al. (2018) for studies focusing on the effect of broadband adoption on growth, Gómez-Barroso and Marbán-Flores (2020) for a literature review on telecommunications more generally, and Xu et al. (2019) who instead focus more specifically on access to the internet as a determinant of innovation. In line with this paper, Mack and Rey (2014) report a generally positive relationship between broadband adoption and the level of knowledge intensive activities across US metropolitan areas but also that specialization in traditional manufacturing has a negative impact on this relationship; Chen et al. (2020) find that high-speed internet significantly increases productivity, but the effect is stronger for the more educated workers (but see Maurseth, 2018, who finds the opposite effect by extending the period of analysis).

cessful in reducing distances, possibly giving a boost to the pre-existing phenomenon towards a digitalization of communications. In terms of the model, this takes the form of a permanent fall in the cost of distance between the two areas d, which implies a strengthening of inter-area spillovers between researchers that the more productive area is more able to exploit. This has the following long-term effects on the balanced growth path:

Proposition 3. *A permanent reduction in the distance between areas, d, determines an increase in the growth rate of the system along the balanced growth path, an increase in the total number of researchers, and a strengthening of the previously existing pattern of specialization.*

Proof. See Appendix A.1 □

Not surprisingly, an improvement in the flow of tacit knowledge has a positive effect on the growth rate of the economy, since both areas essentially benefit from an increase in the effectiveness of their own research efforts. Moreover, a skilled worker becomes relatively more productive if employed in the research sector than in the manufacturing sector, thus causing an influx of these workers from manufacturing to research. However, the relatively more research-intensive area was already better equipped to exploit these increased interactions and thus attracts a larger share of these added researchers. In equilibrium, this same area must also experience a relatively greater reduction of skilled workers in the manufacturing sector and an outflow of unskilled workers towards the relatively more manufacturing-intensive area. This reallocation of workers across sectors and areas strengthens the previously existing patterns of specialization in research and manufacturing, with important repercussions in terms of inter-area inequality.

Corollary 3.1. *A permanent reduction in the distance between areas, d, increases the previously existing differences in the levels of GDP per worker.*

Proof. This follows directly from Corollary 2.1 and Proposition 3. □

Corollary 3.2. *If skilled workers are sufficiently scarce, a permanent reduction in the distance between areas, d, increases the previously existing differences in the areas' Gini coefficients.*

Proof. This follows directly from Corollary 2.2 and Proposition 3. □

Finally, we look at the overall level of inequality, as measured by the Gini coefficient of the entire system,

$$G = \frac{Hw_h}{Y} - \frac{H}{L+H} = \frac{H}{wL+H} - \frac{H}{L+H}, \tag{23}$$

where $w \equiv w_l/w_h = (\alpha/\beta)\,(H_m/L)$. The permanent reduction in the distance between the areas modifies the relative marginal productivity of the workers in the different sectors to the advantage of the researchers (and thus of the skilled workers in general). Together with the strengthening of the previously existing patterns of specialization, this implies the following:

Corollary 3.3. *A permanent reduction in the distance between areas, d, increases the Gini coefficient of the entire system.*

Proof of Corollary 3.3. The Gini coefficient in (23) is clearly decreasing in $w \equiv w_l/w_h = (\alpha/\beta)\,(H_m/L)$. From Proposition 3, $\partial H_m/\partial d > 0$ and thus $\partial G/\partial d < 0$. □

6.2 A Numerical Example

We now report the results of a simple quantitative example to highlight the effects of the shock analysed above on the equilibrium allocation, rather than providing a comprehensive quantitative evaluation. Appendix A.3 presents results following changes in the relative productivity of a research area.

6.2.1 Parameter Choices

A period in our model corresponds to one year. We take $\alpha = \beta = 1/3$, so that the shares of unskilled and skilled labour in production are approximately 33% and the share of income spent on machines is approximately equal to the share of capital. The constant relative risk aversion parameter is taken to be $\sigma = 2$ (see e.g. Kaplow, 2005) and the concavity parameter of the innovation production function is $\eta = 0.5$ (Hall and Ziedonis, 2001). We set $\psi = \psi_1 = \psi_2 = 1 - \eta = 0.5$. The fraction of skilled workers is chosen such that it equals the percentage of individuals in the U.S. with at least a postgraduate degrees i.e. $H/L = 13\%$ (U.S. Census, 2018). We normalise the size of the entire population to ten and $d = 1$. We calibrate the function $\nu_i = d^{-\delta_i/\delta_j}$, which respects Assumption 5 but makes it explicit that what matters is the relative productivity of a research sector rather than its absolute productivity. We set δ_i as to target a long-run annual growth rate equal to 2%; by setting the annual subjective discount rate equal to $\rho = 0.01$, we obtain a long-run annual interest rate equal to $r = 5\%$. Finally, we set the productivity gap between the two research sectors equal to $\delta_j/\delta_i = 75\%$.[10]

6.2.2 Results

The balanced growth path values resulting from the above parametrization are shown in the first column of Table 1. Consistently with the results from the theoretical model, area i, endowed with a relatively more productive research sector, hosts a larger share of researchers than area j (approximately 1.8 times as much), which is instead specialized on manufacturing. As a consequence, area i enjoys a higher level of output per capita but a relatively more unequal society.

Table 1: Balanced Growth Path Values

	Baseline	$\Delta d = -25\%$
g	2.00%	2.40%
$H_{r,i}/H_{r,j}$	177.78%	210.26%
H_r/H	11.76%	12.12%
$H_{r,i}/H_r$	64.00%	67.77%
L_i/L	36.00%	32.23%
y_i/y_j	106.05%	108.38%
G_i/G_j	107.87%	110.79%
G	0.416	0.417

In the second column, we show how the balanced growth path values

[10] For what concerns this exercise, which focuses on the *relative* concentration of workers across areas and aims at providing qualitative results, the values of ϕ_r and ϕ_m are not important, given the endogenous equalization of the external effects in the manufacturing sector. For simplicity, we set $\phi_r = \phi_m = 0$, thus $S_{m,i} = S_{m,j} = 1$; as a consequence, condition (17) simplifies to $H_{r,i}/H_{r,j} = L_j/L_i$. Moreover, we normalize the initial level of the technology frontier to $A = 1$.

change after a permanent negative shock to d, such that the cost of distance between the two areas is reduced by one fourth. Consistently with the theoretical results above, the annual growth rate grows by 20%, since both areas benefit from an increase in the effectiveness of their own research efforts. The shock means that researchers are now relatively more productive than before, and thus the percentage of skilled workers employed in research increases by half a percentage point. However, area i is more equipped to take advantage of this increase, and this strengthens the pre-existing agglomeration dynamics: the share of researchers employed in area i sharply increases, whereas the reverse happens for skilled and unskilled workers in the manufacturing sector. The mass of unskilled workers moving from the research-intensive area to the manufacturing-intensive area is relatively bigger than the mass of skilled workers moving in the opposite direction, causing a relative increase in the level of GDP per worker and the Gini coefficient in area i with respect to area j, and a rise in inequality in the economy at large.

6.2.3 Transitional Dynamics

It is straightforward to see that our expanding variety model does not exhibit transitional dynamics, as the economy always grows at the constant rate given in Proposition 1. Therefore, following an exogenous shock as the one considered in this section, the economy immediately moves to the new balanced growth path. We introduce transitional dynamics into this numerical example by assuming that workers relocate across sectors and areas according to a logistic function.[11]

In particular, we assume that, following a shock, the stock of workers in a given sector, say $H_{r,i}$, evolves according to

$$H_{r,i}(t) = \frac{H^{\star\star}_{r,i} - H^{\star}_{r,i}}{1 + e^{a_{r,i} - t}} + \min\left(H^{\star\star}_{r,i} - H^{\star}_{r,i}\right), \tag{24}$$

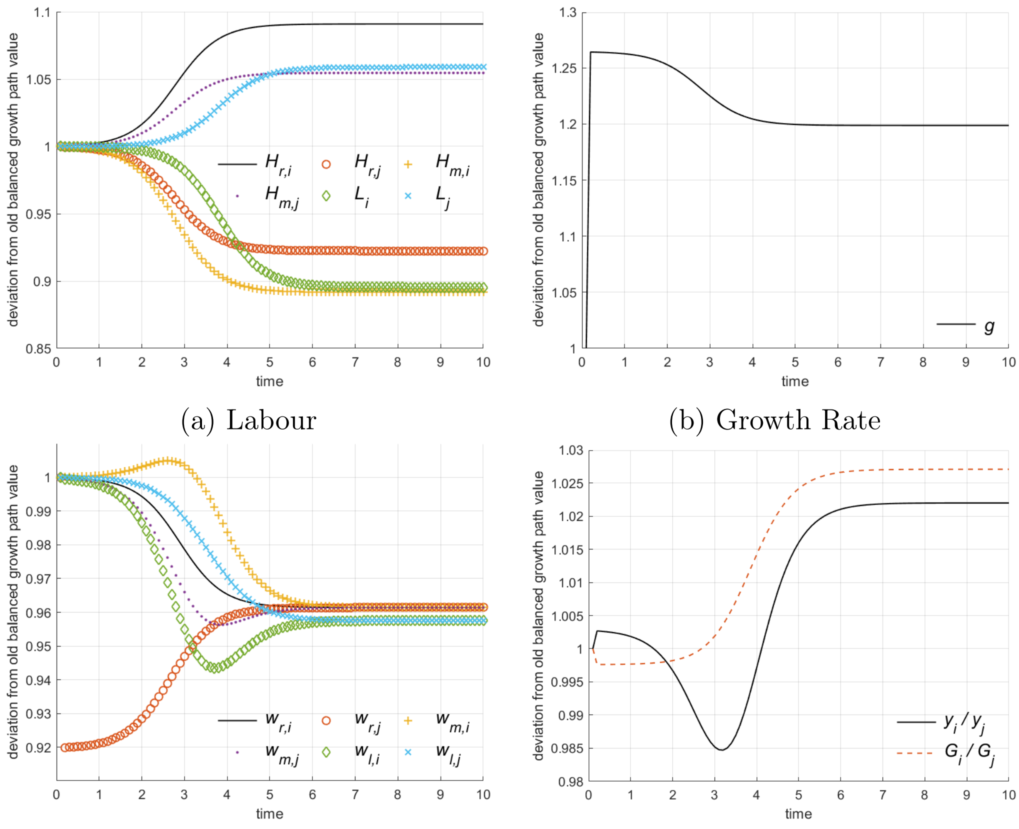
where $H^{\star}_{r,i}$ is the old balanced growth path value, $H^{\star\star}_{r,i}$ is the new balanced

[11] Logistic functions are commonly used to model e.g. population growth (since Verhurst, 1845), migration patterns (e.g. à la Bass, 1969, even if the Bass model was originally built to study the diffusion of new durable products), and the diffusion of innovations (e.g. Griliches, 1957).

growth path value (the *carrying capacity* of the sector), and $a_{r,i}$ is a parameter defining the sigmoid's midpoint. Whereas the initial and final values for each sector correspond to the different balanced growth paths from the numerical examples above, the parameters a remain to be set. To facilitate the interpretation of the results, we assume only two possible values for this parameter: $a_h = 0.3$ for skilled workers and $a_l = 0.4$ for unskilled workers, thus assuming that unskilled workers move more sluggishly in response to shocks (see e.g. Wozniak, 2010, Notowidigdo, 2020). We assume that, along these dynamic paths between balanced growth equilibria, the remaining endogenous variables evolves following the changes in labour stocks according to their respective equations in Sections 4 and 5.

Imagine our economy in period $t = 0$ in the balanced growth path described by the first column of Table 1 being hit by a permanent shock such that $\Delta d = -25\%$; as we already know, this economy will converge to the balanced growth path described by the second column of Table 1. The transitional dynamics are given in Figure 3, where panel 3a presents the evolution of the stocks of labour as deviation from their old balanced growth path values (these are s-shaped as typical of the logistic function). These stocks monotonically converge to their new values, characterized by a strengthening of the previously existing patterns of specialization, but unskilled workers are more sluggish in their response to the shock: by assumption, both $L_i(t)/L_i^\star$ and $L_j(t)/L_j^\star$ take longer to converge to their new balanced growth path values than the curves for skilled workers. Panel 3b shows the resulting evolution of the growth rate of the economy, with a discontinuous jump at the time of the shock and a subsequent smooth descent towards its new value.

Panel 3c shows the evolution of wages: the stocks of labour adjusts to the shock at different speed and the composition of the workforce in each area changes between periods during the transition. This is reflected in wage transitions that are not necessarily monotonic. Adjusting workforce composition and non-monotonic wages translate in inequality dynamics that may exhibit cycles, as shown in panel 3d.



(a) Labour (b) Growth Rate

(c) Wages (d) Inequalities

Figure 3: Transitional dynamics following a shock $\Delta d = -25\%$

7 Convergence Dynamics in Patenting

As shown in Section 6, our model predicts that the diffusion of near-instant communication technologies and videoconferencing should have different effects on metropolitan areas characterised by different levels of absorptive capacity or network capital. In particular, our model predicts a sort of club convergence following such a positive technology shock, whereby areas characterised by a less productive research sector will diverge from areas characterised by a more productive research sector. In this section, we present some empirical evidences in support of this theoretical finding.

7.1 The Distribution Dynamics Approach

We use the distribution dynamics approach (Quah, 1993a,b, 1996a,b, 1997), in which the evolution of the cross-sectional distribution is examined directly, using stochastic kernels to describe both the change in the distribution's external shape and the intra-distribution dynamics.[12]

In simple terms, this works as follows. Let X_t represent a random process, and $F_t(x)$ the corresponding distribution evolving in continuous time, with each F_t defined on the real line; further assume that the distribution at time t admits a density $f_t(x)$. Assuming that the dynamics of f can be modeled as a first order process, the density at some future time $t+s$ is given by

$$f_{t+s}\left(x'\right) = \int_{-\infty}^{\infty} g_s\left(x'|x\right) f_t\left(x\right) dx, \tag{25}$$

where $g_s\left(x'|x\right)$ is the s-period ahead density of x' conditional on x. Specifically, the conditional density function (25) maps the density at time t into the density at time $t+s$ and therefore provides information both on the evolution of the external shape of the distribution and on intra-distributional dynamics between time t and time $t+s$.

To analyze the role an external variable might have on the evolution of the distribution, Quah (1996a, 1997) proposes a conditioning scheme. In particular, given a set of economies $\mathcal{I}$, the scheme is a collection of triples, one for each economy $i \in \mathcal{I}$ at t, where each triple is made of i) an integer lag τ, with $\tau \geq 0$, ii) a subset $C_i(t)$ of $\mathcal{I}$, identifying the collection of economies which are in some form of functional association at time $t-\tau$ with economy i, and iii) a set of probability weights $\omega_i(t)$ for each subset $C_i(t)$. Observations in the conditioned version of the analyzed variable z are obtained normalizing each unit's observation by the weighted average of values in functionally related units. Consequently, the effect of a theoretically motivated factor on the dynamics observed between t and $t+s$ is studied comparing the estimate of the conditional density mapping $f_t(x)$ in $f_{t+s}\left(x'\right)$ with the estimate of the conditional density mapping $f_t(x)$ in $f_{t+s}\left(z'\right)$, where z' is the conditioned version of x'. Differences in the estimated densities are attributed to the role of the conditioning factor.

[12] There are two main approaches to the analysis of convergence, namely the regression approach and the distribution dynamics approach. We opt for the latter because the former does not provide information about what happens to the entire cross-sectional distribution of economies, in terms of both external shape and intra-distributional dynamics. For the relative merits of the two approaches, see e.g. Durlauf and Quah (1999), Temple (1999), Islam (2003), Magrini (2004, 2009), Abreu et al. (2005), Durlauf et al. (2005).

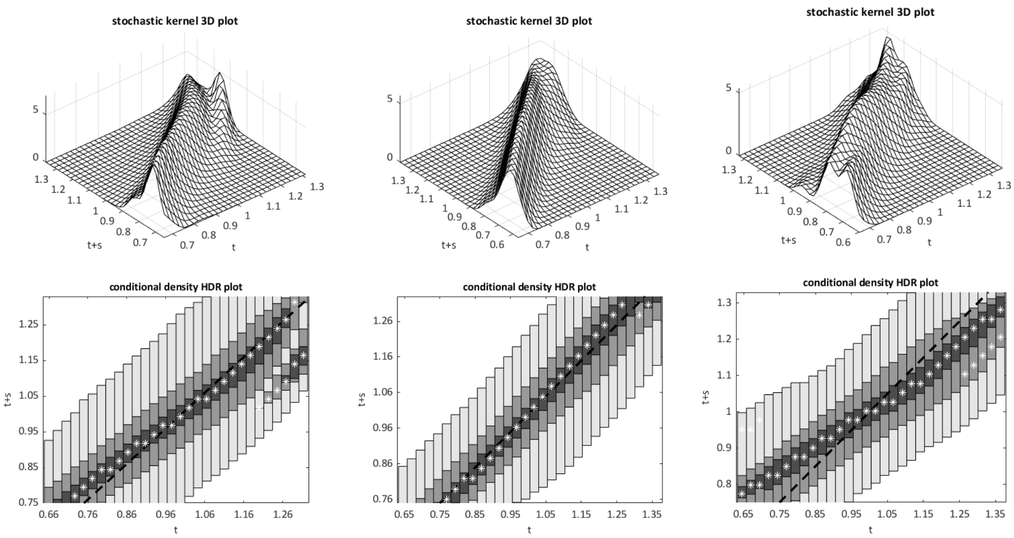
7.2 Dynamics in Patenting Activities

Our model predicts increasing differences in the innovative activities of the two areas following a positive technology shock; thus, here we investigate the evolution of patents per capita across MSAs in the last 50 years.

The dynamics of the distribution of patents per capita among MSAs between 1976 and 2000 are presented in Figure 4a, whereas Figure 4b focuses on the 2000-2019 period. The upper panels show how the cross-sectional distribution at time t evolves into that at time $t + s$, as in Quah (1997). Indeed, the dynamics of the distribution can be analyzed directly from the shape of a plot of the conditional density estimate. When most of the mass is distributed along the 45-degree line, the distribution exhibits persistence; conversely, clockwise (cf. counter-clockwise) rotations highlight a tendency towards convergence (cf. divergence). The lower panels provide a complementary analysis by showing the corresponding highest density region plot (Hyndman, 1996): once again, the 45-degree line highlights persistence properties, whereas a counter-clockwise (cf. clockwise) rotation of the estimated probability mass from the diagonal indicates that divergence (cf. convergence) occurs.

Figure 4a indicates the existence of a tendency toward convergence over the first period, through an evident clockwise rotation with respect to the main diagonal. On the contrary, there seems to be a switch towards divergence in the new century, as illustrated in Figure 4b where an evident counter-clockwise rotation is observable for MSAs at almost all initial levels of patents per capita.

However, our aim is to detect the influence of each area's ability to access and share information over the evolution of its relative position: we thus now resort to a condition scheme. To characterise metropolitan areas in terms of their ability to exploit the positive technology shocks, one likely candidate would be broadband availability, since broadband and related technologies lower the cost of sending and receiving information. Yet, this would raise the spectre of endogeneity, as broadband availability and economic growth are likely to influence each other. We thus follow Kolko (2012) and instrument broadband availability with the topography



(a) 1976-2000 (b) 2000-19 (c) 2000-19 conditioned

Figure 4: Distribution of Patents Per Capita Across MSAs

Notes. Each upper panel shows the evolution of the cross-sectional distribution of patents per capita across MSAs between the initial and the final period; the lower panels show the correspondig highest density region plots (Hyndman, 1996) where vertical bands represent the projection of the conditional density of patents per capita at time t+s on patents per capita at time t. In each band the 25% (the darkest-shaded region), 75%, 99% HDRs are reported. The dashed line is the 45-degree line and the asterisks represent the modes). Data are normalised as to have mean one. Own elaborations using data from USPTO, DOI, and USDA. Only MSAs for which we observe patents, population, topography scale, and amenities scale are included (for a total of 344 MSAs per year).

of the area. In particular, we use the topography scale by the National Atlas of the United States of America of the U.S. Department of Interior: this classifies areas according to the twenty-one land-form units specified by Hammond (1964), which are based on the slope of the terrain, the difference between its maximum and minimum elevation, and its profile type (i.e. the percentage of gently sloping terrain laying below or above the window's average elevation).[13]

To be a good instrument, the topography scale should be correlated

[13] The topography scale is available at the county level. We thus calculate the topography scale of each MSA as a weighted average of the topography scale of its current counties, where the weights are the relative land area in square miles as provided by the 1990 U.S. Census. We use definitions provided by the National Bureau of Economic Research, `https://www.nber.org/research/data/census-core-based-statistical-area-cbsa-federal-information-processing-series-fips-county-crosswalk`.

with broadband, and indeed Kolko (2012) shows that broadband availability is strongly negatively correlated with slope of terrain, likely because infrastructure is more expensive to deploy in areas with steeper terrain. At the same time, topography should not be independently correlated with patenting activity, which might not be satisfied (e.g. Rosenthal and Strange, 2008, Saiz, 2010). For example, steeper areas might enjoy lower wages and higher employment if skilled workers value steeper terrains for recreational activities or as an amenity offering views. To partially control for this, we divide the topography scale by the natural amenities scale provided by the U.S. Department of Agriculture,[14] which provides a measure of the physical characteristics of an area that enhance the location as a place to live (e.g. temperatures and humidity in winter and summer and percentage of surface covered in water). Our conditioning scheme is then as follows: we first measure the maximum distance in terms of our modified scale of topography in 1990 between any two MSAs in our dataset, and then, for each MSA $i \in \mathcal{I}$, we let $C_i(t)$ be the set of MSAs within 10% of this maximum distance to i; on average, a MSA has 35 "neighbours". We then weight MSAs in $C_i(t)$ uniformly.

The panels of Figure 4c display the cross-section distribution and the highest density region plot between observed values of patents per capita in 2000 and topography-conditioned values of patents per capita in 2019. These differ markedly from the unconditioned plots in Figure 4b, since now they exhibit a strong clockwise rotation. This supports the existence of convergence clubs on the basis of the conditioning factor: it appears that the polarization earlier identified in the unconditional distribution-dynamics of patents per capita across MSAs is partly explained by our measure of topography (corrected for natural amenities).

8 Conclusions

Broadband technology and high-speed connections have steadily changed the way people lived and worked over the last decades. We proposed an

[14] See USDA, Economic Research Service, available at `https://www.ers.usda.gov/data-products/natural-amenities-scale/`. This is provided for counties, so we aggregate it at the MSA level using relative land surfaces as for the topography scale.

endogenous growth model with two areas to investigate how this could have changed the spatial distribution of research activities and their contribution to growth, and the subsequent repercussions on per capita income and inequality levels. We showed that, when one area is endowed with an higher ability to assimilate new knowledge and apply it to commercial use, relative specialization arises in equilibrium, as this area attracts a larger share of researchers; conversely, the other area specializes in manufacturing activities. Since researchers are scarcer in the entire population and command a higher wage than the average manufacturing worker, relative specialization in research translates into a higher income per capita level but a more unequal income distribution.

In this context, a boost towards a digitalization of communications increases the growth rate of the overall economy, but also strengthens the previously existing patterns of specialization, thus increasing disparities in income per capita and Gini coefficients between areas, as well as the Gini coefficient of the entire system. These results are consistent with empirical evidences pointing to increases in the concentration of innovative activities, economy-wide income inequality, and skills and income divergence across urban areas experienced by the United States in the last decades. This may also have important implications for the post-pandemic future, as digital communication will most probably be integral part of daily working to a much higher extent than before.[15] Our results suggest that this may lead to a further rise of specialization, agglomeration, and inequalities across

[15] For example, the proportion of US employees who primarily work from home tripled from 0.75% in 1980 to 2.4% in 2010 (Bloom et al., 2015), but this number was an order of magnitude larger in March 2020, when 42% of respondents to a survey of American adults who earned at least $20,000 in labour income in 2019 were working from home (see `https://voxeu.org/article/covid-19-and-labour-reallocation-evidence-us`). Even if some of these jobs will go back to be performed in offices, it is likely that working remotely will still be part of the new reality: for example, Dingel and Neiman (2020) estimate that 37% of jobs in the US can be performed entirely at home, many tech giants have already made working from home a permanent option for employees (see the article on Business Insider by Aaron Holmes, `https://www.businessinsider.com/how-tech-companies-plan-to-reopen-facebook-google-microsoft-amazon-2020-5?IR=T`) and the share of working days spent at home is expected to triple after the Covid-19 crisis ends compared to before the pandemic hit (see Barrero et al., 2021, and the article by Altig et al. for the Federal Reserve Bank of Atlanta's Policy Hub: Macroblog, `https://www.frbatlanta.org/blogs/macroblog/2020/05/28/firms-expect-working-from-home-to-triple`).

areas.

However stylised, our model predicts that policies aimed at facilitating the diffusion of knowledge across the entire system (e.g. improving broadband access) increase the growth rate of the economy, but the resulting strengthening of the previously existing patterns of specialization leads to more inequality within and across areas. Place-based policies aimed towards increasing the productivity of an area's research system similarly imply a trade-off between growth and equality, but results may be significantly different depending on the targeted area. In particular, policies aimed at the more backward research sector increase the growth rate and the Gini coefficient of the entire system (by widening the wage gap between skilled and unskilled workers) but reduce relative specialization and thus differences in income per capita across areas.

We made many simplifying assumptions to keep the model tractable. For example, we have assumed that one area is exogenously endowed with a more productive research sector; it would also be interesting to analyse the case in which this is the outcome of conscious investments in network capital and absorptive capacity (as suggested by e.g. Huggins and Thompson, 2014). Moreover, to focus primarily on the knowledge externality, we have assumed zero transport costs and no differences in the areas' amenities; however, one could include those to analyse how workers and firms balance these factors in making location decisions. Finally, by adding more areas, one could study the effects of reduction in the "cost of distance" on the centrality of a research sector. We leave these extensions to future research.

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A Appendix (For Online Publication)

A.1 Proofs

Proof of Equation (17). We first prove that $w_{l,i}/w_{l,j} = w_{m,i}/w_{m,j} = 1$ implies $L_i/L_j = H_{m,i}/H_{m,j}$. Substitute equations (16a) and (16b) into $w_{l,i}/w_{l,j} = w_{m,i}/w_{m,j} = 1$ to obtain

$$\frac{\alpha L_i^{\alpha-1} H_{m,i}^{\beta} A x_i^{\gamma} S_{m,i}}{\alpha L_j^{\alpha-1} H_{m,j}^{\beta} A x_j^{\gamma} S_{m,j}} = \frac{\beta L_i^{\alpha} H_{m,i}^{\beta-1} A x_i^{\gamma} S_{m,i}}{\beta L_j^{\alpha} H_{m,j}^{\beta-1} A x_j^{\gamma} S_{m,j}} = 1.$$

Trivial algebraic steps lead to

$$\begin{aligned} \frac{L_j}{L_i} = \frac{H_{m,j}}{H_{m,i}} &= \left(\frac{L_j}{L_i}\right)^{\alpha} \left(\frac{H_{m,j}}{H_{m,i}}\right)^{\beta} \left(\frac{x_j}{x_i}\right)^{\gamma} \frac{S_{m,j}}{S_{m,i}} \\ &= \left(\frac{L_j}{L_i}\right)^{\alpha+\beta} \left(\frac{x_j}{x_i}\right)^{\gamma} \frac{S_{m,j}}{S_{m,i}}. \end{aligned} \tag{A.1}$$

Note that $H_{m,i}/L_i = H_{m,j}/L_j = H_m/L$, where $H_m \equiv H_{m,i} + H_{m,j}$ is the aggregate number of skilled workers employed in the manufacturing sector. The ratio of the demand functions of intermediate input can be computed from (11) as

$$\frac{x_i}{x_j} = \left(\frac{L_i}{L_j}\right)^{\frac{\alpha+\beta}{1-\gamma}} \left(\frac{S_{m,i}}{S_{m,j}}\right)^{\frac{1}{1-\gamma}} = \frac{L_i}{L_j} \left(\frac{S_{m,i}}{S_{m,j}}\right)^{\frac{1}{1-\gamma}}, \tag{A.2}$$

where we used the first equality of (A.1) and $\alpha + \beta = 1 - \gamma$. Substituting this into (A.1), one obtains

$$\frac{L_j}{L_i} = \left(\frac{L_j}{L_i}\right)^{\alpha+\beta+\gamma} \left(\frac{S_{m,i}}{S_{m,j}}\right)^{\frac{\gamma}{1-\gamma}+1} \tag{A.3}$$

which implies $S_{m,i} = S_{m,j}$ in equilibrium, and thus $L_i/L_j = x_i/x_j$ from (A.2). Substitute the functional forms from (5) in $S_{m,i} = S_{m,j}$ to obtain Equation (17). □

Proof of equation (18). The ratio of the wages of skilled workers across sectors is

$$\begin{aligned} \frac{w_m}{w_r} &= \frac{\beta L_i^{\alpha} H_{m,i}^{\beta-1} A x_i^{\gamma} S_{m,i}}{A X \eta \delta_i H_{r,i}^{\eta-1} S_{r,ij} (1-\gamma) r^{-1}} = \frac{\beta L_i^{\alpha} H_{m,i}^{\beta-1} x_i^{\gamma} S_{m,i} r}{X \eta g H_r^{-1} (1-\gamma)} = \frac{\beta H_{m,i}^{-1} x_i r \gamma^{-1}}{X \eta g H_r^{-1} (1-\gamma)} = \\ &= \frac{\beta x_i H_r r}{X \eta g \gamma (1-\gamma) H_{m,i}} = \frac{\beta x_i H_r r}{x_i H_m H_{m,i}^{-1} \eta g \gamma (1-\gamma) H_{m,i}} = \frac{\beta H_r r}{H_m \eta g \gamma (1-\gamma)}, \end{aligned}$$

where the first equality in the first line comes from simply replacing wages with their definition in (14) and (16b), the second from applying (20) to the denominator, and the third by applying (11) to the numerator. The first equality in the second line comes from rearranging the previous expression, the second from using $X = x_i + x_j$ combined with $x_i/x_j = H_{m,i}/H_{m,j}$ to substitute for X, and the third from simply rearranging the previous one. Finally, it is enough to notice that w_m/w_r must be equal to one in equilibrium to obtain expression (18). □

Proof of Proposition 1. Equation (20) is obtained by substituting (7) and (9) in $g \equiv (\dot{A}_i + \dot{A}_j)/A$ and then using condition (15). The preceding discussion establishes most of the claims in the proposition, except that abstract knowledge grows at the same rate in both urban areas and that the equilibrium is stable. To determine the growth rates of abstract knowledge within each urban area, $g_i(t) \equiv \dot{A}_i(t)/A_i(t)$ and $g_j(t) \equiv \dot{A}_j(t)/A_j(t)$, it will be convenient to define $\mathcal{A}_i(t) \equiv A_i(t)/A(t)$ as an inverse measure of the proportional abstract knowledge gap between area i and the overall economy. Applying logs to both sides and taking derivatives with respect to time, we obtain $g_i(t) = \dot{\mathcal{A}}_i(t)/\mathcal{A}(t) + g$, or, equivalently,

$$g_i(t) = \delta_i S_{r,ij} H_{r,i}^{\eta} \frac{A(t)}{A_i(t)}. \tag{A.4}$$

Since along the balanced growth path $g_i(t)$ must be constant, $A_i(t)$ must grow at the constant rate g; with an identical reasoning, also $A_j(t)$ grows at g. Alternatively, one can use (15) to show that $\dot{A}_i/\dot{A}_j = H_{r,i}/H_{r,j}$ in equilibrium. Applying L'Hôpital's rule, $\lim_{t\to\infty} A_i/A_j = H_{r,i}/H_{r,j}$ and $\lim_{t\to\infty} A/A_i = H_r/H_{r,i}$. Substituting the last result into (A.4), one obtains $g_i(t) = g$.

To check the stability of the equilibrium, it is sufficient to see whether the wage gap declines as researchers move towards the area offering a higher wage. The condition for the stability of the equilibrium in equation (15) thus is

$$\left.\frac{\partial w_{r,i}}{\partial H_{r,i}}\right|_{H_{r,i}=H^{\star}_{r,i}} < 0, \tag{A.5}$$

where $H^{\star}_{r,i}$ makes it explicit that we are evaluating the labour allocation along the balanced growth path. Using equation (14), the definition of the spatial spillovers in equation (9), and taking note that $H_{r,j} = H_r - H_{r,i}$, equilibrium stability requires

$$\left(\frac{\psi_1 + \eta - 1}{H_{r,i}} - \frac{\psi_2}{H_{r,j}}\right)\left(H_{r,i}^{\psi_1+\eta-1} H_{r,j}^{\psi_2}\right)\nu_i^{\psi}\eta\delta_i X \frac{1-\gamma}{r} A < 0.$$

The sign of the left hand side depends on the sign of its first term; hence, the stability condition simplifies to:

$$\frac{\psi_1 + \eta - 1}{\psi_2} < \frac{H_{r,i}}{H_{r,j}}. \tag{A.6}$$

When $\delta_i > \delta_j$ as in the rest of the analysis, the equilibrium allocation leads to $H_{r,i}/H_{r,j} > 1$; this condition is then always satisfied given Assumption 4. □

Proof of Corollary 2.1. Letting $w \equiv w_l/w_h$,

$$\begin{aligned}
\frac{y_i}{y_j} &\equiv \frac{w_l L_i + w_h H_{m,i} + w_h H_{r,i}}{w_l L_j + w_h H_{m,j} + w_h H_{r,j}} \left(\frac{L_j + H_{m,j} + H_{r,j}}{L_i + H_{m,i} + H_{r,i}}\right) = \\
&= \frac{L_i w_h \left(w + H_{m,i}/L_i + H_{r,i}/L_i\right)}{L_j w_h \left(w + H_{m,j}/L_j + H_{r,j}/L_j\right)} \left(\frac{L_j \left(1 + H_{m,j}/L_j + H_{r,j}/L_j\right)}{L_i \left(1 + H_{m,i}/L_i + H_{r,i}/L_i\right)}\right) = \\
&= \frac{w + H_{m,i}/L_i + H_{r,i}/L_i}{w + H_{m,j}/L_j + H_{r,j}/L_j} \left(\frac{1 + H_{m,j}/L_j + H_{r,j}/L_j}{1 + H_{m,i}/L_i + H_{r,i}/L_i}\right) = \\
&= \frac{w + H_m/L + H_{r,i}/L_i}{w + H_m/L + H_{r,j}/L_j} \left(\frac{1 + H_m/L + H_{r,j}/L_j}{1 + H_m/L + H_{r,i}/L_i}\right) = \\
&= \frac{\left(w + \frac{H_m}{L}\right)\left(1 + \frac{H_m}{L}\right) + \frac{H_{r,i}}{L_i}\left(1 + \frac{H_m}{L} + \frac{H_{r,j}}{L_j}\right) + \left(w + \frac{H_m}{L}\right)\frac{H_{r,j}}{L_j}}{\left(w + \frac{H_m}{L}\right)\left(1 + \frac{H_m}{L}\right) + \frac{H_{r,j}}{L_j}\left(1 + \frac{H_m}{L} + \frac{H_{r,i}}{L_i}\right) + \left(w + \frac{H_m}{L}\right)\frac{H_{r,i}}{L_i}},
\end{aligned}$$

where the first line follows from dividing (21) by total area's employment, the second from aggregating for $w_h L_i$ and $w_h L_j$ and rewriting, the third from simplifying, the fourth from using the following identities, $H_{m,i}/L_i = H_{m,j}/L_j = H_m/L$, and the last by multiplying throughout. A sufficient condition for $y_i > y_j$ then is

$$\frac{H_{r,i}}{L_i}\left(1 + \frac{H_m}{L}\right) + \left(w + \frac{H_m}{L}\right)\frac{H_{r,j}}{L_j} > \frac{H_{r,j}}{L_j}\left(1 + \frac{H_m}{L}\right) + \left(w + \frac{H_m}{L}\right)\frac{H_{r,i}}{L_i}$$

$$\text{i.e.} \quad \left(\frac{H_{r,i}}{L_i} - \frac{H_{r,j}}{L_j}\right)(1 - w) > 0.$$

Substitute (16a) and (16b) into $w \equiv w_l/w_h$ to obtain $w = (\alpha/\beta)(H_m/L)$, which is lower than one by Assumption 3. Then, this sufficient condition is always satisfied since $H_{r,i}/H_{r,j} > L_i/L_j$ by Proposition 2. □

Proof of Corollary 2.2. The Gini coefficient in area i is given in equation (22); equivalently $G_i = H_i/(H_i + L_i w) - H_i/(H_i + L_i)$, where $H_i \equiv H_{r,i} +$

$H_{m,i}$. Then, with some analytical steps,

$$\begin{aligned} G_i - G_j &= \left(\frac{H_i}{H_i + L_i w} - \frac{H_j}{H_j + L_j w} \right) - \left(\frac{H_i}{H_i + L_i} - \frac{H_j}{H_j + L_j} \right) \\ &= \frac{(w-1)\,(H_i L_j - H_j L_i)\,(H_i H_j - w L_i L_j)}{(H_i + L_i w)\,(H_j + L_j w)\,(H_i + L_i)\,(H_j + L_j)}. \end{aligned}$$

Since the denominator is positive, $w - 1 < 0$ by Assumption 3, and $H_i L_j - H_j L_i > 0$ by Assumption 6 and Proposition 2, a sufficient condition for $G_i > G_j$ is $H_i H_j - w L_i L_j < 0$. Using $w = (\alpha/\beta)(H_m/L)$ and conditions (15) and (17), this is equivalent to

$$L > \frac{\beta}{\alpha} \frac{1}{H_m} \left\{ H_m H_m + H_m H_r \left[\left(\frac{\delta_i \nu_i^{\psi}}{\delta_j \nu_j^{\psi}} \right)^{-\Psi} + \left(\frac{\delta_i \nu_i^{\psi}}{\delta_j \nu_j^{\psi}} \right)^{\Psi} \right] + H_r H_r \right\},$$

where $\Psi = \phi_r / \left[\phi_m \left(1 - \eta - \psi_1 + \psi_2\right)\right]$. This is always satisfied if L is sufficiently larger than H. □

Proof of Proposition 3. Take the derivative of $H_{r,i}/H_{r,j}$ in (15) with respect to d to obtain

$$\frac{\partial \left(H_{r,i}/H_{r,j}\right)}{\partial d} = \left(\frac{\delta_i}{\delta_j} \right)^{\frac{1}{1-\eta}} \frac{\psi}{1-\eta} \left(\frac{\nu_i}{\nu_j} \right)^{\frac{\psi - 1 + \eta}{1-\eta}} \frac{1}{v_j^2} \left(v_j \frac{\partial v_i}{\partial d} - v_i \frac{\partial v_j}{\partial d} \right),$$

the sign of which depends on the sign of the last term on the right hand side. This is negative given Assumption 5, implying that a reduction in d determines an increase in $H_{r,i}/H_{r,j}$. From the equilibrium condition in (17),

$$\frac{H_{r,i}/L_i}{H_{r,j}/L_j} = \left(\frac{H_{r,i}}{H_{r,j}} \right)^{\frac{\phi_m + \phi_r}{\phi_m}}, \tag{A.7}$$

which is increasing in $H_{r,i}/H_{r,j}$ since $(\phi_m + \phi_r)/(\phi_m) > 0$: the shock thus strengthens the previously existing patterns of specialization.

Along the balanced growth path, the constant growth rate is given by equation (20). Using (9), and depending on which of the two definitions is used, the derivative with respect to d is

$$\begin{aligned} \frac{\partial g}{\partial d} &= g \left[-\frac{1 - \eta - \psi_1}{H_{r,i}} \frac{\partial H_{r,i}}{\partial d} + \frac{\psi_2}{H_{r,j}} \frac{\partial H_{r,j}}{\partial d} + \frac{\psi}{\nu_i} \frac{\partial \nu_i}{\partial d} + \frac{1}{H_r} \frac{\partial H_r}{\partial d} \right] \\ &= g \left[-\frac{1 - \eta - \psi_1}{H_{r,j}} \frac{\partial H_{r,j}}{\partial d} + \frac{\psi_2}{H_{r,i}} \frac{\partial H_{r,i}}{\partial d} + \frac{\psi}{\nu_j} \frac{\partial \nu_j}{\partial d} + \frac{1}{H_r} \frac{\partial H_r}{\partial d} \right]. \end{aligned} \tag{A.8}$$

Substituting the following results obtained from differentiating (18),

$$\frac{\partial H_r}{\partial d} = \frac{(H - H_r)H_r}{H}\frac{\rho}{rg}\frac{\partial g}{\partial d}, \tag{A.9}$$

into (A.8), and rearranging, one obtains

$$\begin{aligned} \frac{\partial g}{\partial d} &= \frac{rHg}{rH - (H - H_r)\rho}\left[-\frac{1-\eta-\psi_1}{H_{r,i}}\frac{\partial H_{r,i}}{\partial d} + \frac{\psi_2}{H_{r,j}}\frac{\partial H_{r,j}}{\partial d} + \frac{\psi}{\nu_i}\frac{\partial \nu_i}{\partial d}\right] \\ &= \frac{rHg}{rH - (H - H_r)\rho}\left[-\frac{1-\eta-\psi_1}{H_{r,j}}\frac{\partial H_{r,j}}{\partial d} + \frac{\psi_2}{H_{r,i}}\frac{\partial H_{r,i}}{\partial d} + \frac{\psi}{\nu_j}\frac{\partial \nu_j}{\partial d}\right], \end{aligned} \tag{A.10}$$

which requires an equalisation of the terms inside the brackets. After some manipulations, this accounts to

$$\frac{\partial H_{r,j}}{\partial d}H_{r,j}^{-1} - \frac{\partial H_{r,i}}{\partial d}H_{r,i}^{-1} = \frac{\psi}{1-\eta-\psi_1+\psi_2}\left(\frac{\partial \nu_j}{\partial d}\nu_j^{-1} - \frac{\partial \nu_i}{\partial d}\nu_i^{-1}\right). \tag{A.11}$$

The right hand side of (A.11) is positive by Assumptions 4 and 5. As a consequence,

$$\frac{\partial H_{r,j}}{\partial d} > \frac{\partial H_{r,i}}{\partial d}\frac{H_{r,j}}{H_{r,i}}, \tag{A.12}$$

where the last ratio on the right hand side is lower than one by Assumption 6 and Proposition 2. Since $\partial(H_{r,i}/H_{r,j})/\partial d < 0$ was proven above, there are two possible cases consistent with (A.12):

$$i) \quad \frac{\partial H_{r,i}}{\partial d} < \frac{\partial H_{r,j}}{\partial d} < 0 \tag{A.13}$$

$$ii) \quad \frac{\partial H_{r,j}}{\partial d} > 0 > \frac{\partial H_{r,i}}{\partial d}. \tag{A.14}$$

In case i), both research sectors experience an influx of skilled workers after a decrease in d, but the change is relatively bigger in the more advanced research sector. Since $\partial H_r/\partial d < 0$, equation (A.9) implies $\partial g/\partial d < 0$: a permanent negative shock to d causes a permanent increase in the common growth rate g. In case ii), $H_{r,j}$ decreases after a negative shock to d, whereas $H_{r,i}$ increases. From the second line in (A.10), g increases; from (A.9), so does H_r. □

A.2 An Alternative Form for the Agglomeration Effect

In this section, we quickly show that results are qualitatively similar if the functional form of the agglomeration effect in manufacturing depends on average, rather than total, human capital in the area (similarly to e.g. Lucas, 1988, Black and Henderson, 1999, Moretti, 2004). Let

$$S_{m,i} = \left(\frac{H_{r,i} H_{m,i}}{L_i + H_{m,i} + H_{r,i}} \right)^{\phi}, \tag{A.15}$$

where $H_{r,i}$ represents skilled labour employed in i's research sector and $0 \leq \phi < 1$ determines the strength of the economies arising from the agglomeration of skilled workers; when $\phi = 0$, there are no local spillover effects in the manufacturing sector. Since the denominator is the total number of workers in the area, this form of spillovers implies that each manufacturing sectors enjoys a positive externality effect of the average level of human capital.[16]

In equilibrium, $S_{m,i} = S_{m,j}$, i.e.

$$\frac{H_{r,i}}{H_{r,j}} = \frac{1 + H_m/L + H_{r,i}/L_i}{1 + H_m/L + H_{r,j}/L_j}. \tag{A.16}$$

Under Assumption 6, $H_{r,i} > H_{r,j}$, thus $H_{r,i}/H_{r,j} > L_i/L_j$: there is specialization in equilibrium, which is sufficient to characterize the remaining results.

[16] If the numerator is additive rather than multiplicative, in equilibrium area i attracts a larger population but the composition of the workforce is the same across areas.

A.3 A Productivity Shock

Here, we turn our attention to the long-term effects of a variation in the productivity of a research sector, i.e. δ_i or δ_j.

Proposition A.1. *A permanent reduction in the productivity of an urban area, δ_i or δ_j, determines a decrease in the growth rate of the system along the balanced growth path and a decrease in the total number of researchers. A fall (cf. increase) in the productivity gap between the two research sectors, δ_i/δ_j, determines a weakening (cf. strengthening) of the previously existing pattern of specialization.*

Proof of Proposition A.1. From the equilibrium condition (15), and using Assumption 5,

$$\frac{\partial\left(H_{r,i}/H_{r,j}\right)}{\partial\delta_i} = \frac{1}{1-\eta-\psi_1+\psi_2}\frac{H_{r,i}}{H_{r,j}}\left[\frac{1}{\delta_i}+\frac{\psi}{\nu_i}\frac{\partial\nu_i}{\partial\delta_i}\right] > 0 \tag{A.17a}$$

$$\frac{\partial\left(H_{r,i}/H_{r,j}\right)}{\partial\delta_j} = -\frac{1}{1-\eta-\psi_1+\psi_2}\frac{H_{r,i}}{H_{r,j}}\left[\frac{1}{\delta_j}+\frac{\psi}{\nu_j}\frac{\partial\nu_j}{\partial\delta_j}\right] < 0. \tag{A.17b}$$

Therefore, a decrease in δ_i (δ_j) determines a decrease (increase) in $H_{r,i}/H_{r,j}$; given the equilibrium condition in (17), this is associated with an decrease (increase) in specialization.

Along the balanced growth path, the constant growth rate is given by equation (20). Using (9), and depending on which of the two definitions is used, the derivative with respect to δ_i is

$$\begin{aligned}\frac{\partial g}{\partial\delta_i} &= g\left[\frac{1}{\delta_i}-\frac{1-\eta-\psi_1}{H_{r,i}}\frac{\partial H_{r,i}}{\partial\delta_i}+\frac{\partial H_{r,j}}{\partial\delta_i}\frac{\psi_2}{H_{r,j}}+\frac{\partial\nu_i}{\partial\delta_i}\frac{\psi}{\nu_i}+\frac{\partial H_r}{\partial\delta_i}\frac{1}{H_r}\right]\\ &= g\left[-\frac{1-\eta-\psi_1}{H_{r,j}}\frac{\partial H_{r,j}}{\partial\delta_i}+\frac{\partial H_{r,i}}{\partial\delta_i}\frac{\psi_2}{H_{r,i}}+\frac{\partial H_r}{\partial\delta_i}\frac{1}{H_r}\right].\end{aligned} \tag{A.18}$$

Substituting the following results obtained from differentiating (18),

$$\frac{\partial H_r}{\partial\delta_i} = \frac{\left(H-H_r\right)H_r}{H}\frac{\rho}{rg}\frac{\partial g}{\partial\delta_i}, \tag{A.19}$$

into (A.18), and rearranging, one obtains

$$\begin{aligned}\frac{\partial g}{\partial\delta_i} &= \frac{rHg}{rH-(H-H_r)\rho}\left[\frac{1}{\delta_i}-\frac{1-\eta-\psi_1}{H_{r,i}}\frac{\partial H_{r,i}}{\partial\delta_i}+\frac{\partial H_{r,j}}{\partial\delta_i}\frac{\psi_2}{H_{r,j}}+\frac{\partial\nu_i}{\partial\delta_i}\frac{\psi}{\nu_i}\right]\\ &= \frac{rHg}{rH-(H-H_r)\rho}\left[-\frac{1-\eta-\psi_1}{H_{r,j}}\frac{\partial H_{r,j}}{\partial\delta_i}+\frac{\partial H_{r,i}}{\partial\delta_i}\frac{\psi_2}{H_{r,i}}\right],\end{aligned} \tag{A.20}$$

which requires an equalisation of the terms inside the brackets. After some manipulations, this accounts to

$$(1-\eta-\psi_1+\psi_2)\left\{\frac{\partial H_{r,j}}{\partial \delta_i}H_{r,j}^{-1}-\frac{\partial H_{r,i}}{\partial \delta_i}H_{r,i}^{-1}\right\}=-\left\{\frac{\partial \nu_i}{\partial \delta_i}\frac{\psi}{\nu_i}+\frac{1}{\delta_i}\right\}. \quad \text{(A.21)}$$

The right hand side of (A.21) is negative by Assumptions 4 and 5. As a consequence,

$$\frac{\partial H_{r,j}}{\partial \delta_i}<\frac{\partial H_{r,i}}{\partial \delta_i}\frac{H_{r,j}}{H_{r,i}}, \quad \text{(A.22)}$$

where the last ratio on the right hand side is lower than one by Assumption 6 and Proposition 2. Since $\partial(H_{r,i}/H_{r,j})/\partial\delta_i>0$ as proven above, there are two possible cases consistent with (A.22):

$$i)\quad \frac{\partial H_{r,i}}{\partial \delta_i}>\frac{\partial H_{r,j}}{\partial \delta_i}>0 \quad \text{(A.23)}$$

$$ii)\quad \frac{\partial H_{r,i}}{\partial \delta_i}>0>\frac{\partial H_{r,j}}{\partial \delta_i}. \quad \text{(A.24)}$$

In case i), both $H_{r,i}$ and $H_{r,j}$ decrease after a negative shock to δ_i. Since $\partial H_r/\partial\delta_i>0$, equation (A.19) implies $\partial g/\partial\delta_i>0$: a permanent negative shock to δ_i causes a permanent decrease in the common growth rate g. In case ii), $H_{r,i}$ decreases after a negative shock to δ_i, whereas $H_{r,j}$ increases. From the second line in (A.20), g decreases; from (A.19), so does H_r, which implies a rise in the number of skilled workers in the manufacturing sectors.

Now, we turn our attention to the long-term effects of an increase in the productivity of area j's research sector (thus determining a fall in the productivity gap). From (A.17b), an increase in δ_j determines a decrease in $H_{r,i}/H_{r,j}$; given the equilibrium condition in (17), this is associated with an decrease in specialization.

Along the balanced growth path, the constant growth rate is given by equation (20). Using (9), and depending on which of the two definitions is used, the derivative with respect to δ_j is

$$\begin{aligned}\frac{\partial g}{\partial \delta_j}&=g\left[\frac{1}{\delta_j}-\frac{1-\eta-\psi_1}{H_{r,j}}\frac{\partial H_{r,j}}{\partial \delta_j}+\frac{\partial H_{r,i}}{\partial \delta_j}\frac{\psi_2}{H_{r,i}}+\frac{\partial \nu_j}{\partial \delta_j}\frac{\psi}{\nu_j}+\frac{\partial H_r}{\partial \delta_j}\frac{1}{H_r}\right]\\&=g\left[-\frac{1-\eta-\psi_1}{H_{r,i}}\frac{\partial H_{r,i}}{\partial \delta_j}+\frac{\partial H_{r,j}}{\partial \delta_j}\frac{\psi_2}{H_{r,j}}+\frac{\partial H_r}{\partial \delta_j}\frac{1}{H_r}\right].\end{aligned} \quad \text{(A.25)}$$

Substituting the following results obtained from differentiating (18),

$$\frac{\partial H_r}{\partial \delta_j}=\frac{(H-H_r)\,H_r}{H}\frac{\rho}{rg}\frac{\partial g}{\partial \delta_j}, \quad \text{(A.26)}$$

into (A.25), and rearranging, one obtains

$$\frac{\partial g}{\partial \delta_j} = \frac{rHg}{rH - (H - H_r)\rho}\left[\frac{1}{\delta_j} - \frac{1-\eta-\psi_1}{H_{r,j}}\frac{\partial H_{r,j}}{\partial \delta_j} + \frac{\partial H_{r,i}}{\partial \delta_j}\frac{\psi_2}{H_{r,i}} + \frac{\partial \nu_j}{\partial \delta_j}\frac{\psi}{\nu_j}\right]$$
$$= \frac{rHg}{rH - (H - H_r)\rho}\left[-\frac{1-\eta-\psi_1}{H_{r,i}}\frac{\partial H_{r,i}}{\partial \delta_j} + \frac{\partial H_{r,j}}{\partial \delta_j}\frac{\psi_2}{H_{r,j}}\right], \tag{A.27}$$

which requires an equalisation of the terms inside the brackets. After some manipulations, this accounts to

$$(1-\eta-\psi_1+\psi_2)\left\{\frac{\partial H_{r,i}}{\partial \delta_j}H_{r,i}^{-1} - \frac{\partial H_{r,j}}{\partial \delta_j}H_{r,j}^{-1}\right\} = -\left\{\frac{\partial \nu_j}{\partial \delta_j}\frac{\psi}{\nu_j} + \frac{1}{\delta_j}\right\}. \tag{A.28}$$

The right hand side of (A.21) is negative by Assumptions 4 and 5. As a consequence,

$$\frac{\partial H_{r,i}}{\partial \delta_j} < \frac{\partial H_{r,j}}{\partial \delta_j}\frac{H_{r,i}}{H_{r,j}}, \tag{A.29}$$

where the last ratio on the right hand side is higher than one by Assumption 6 and Proposition 2. Since $\partial(H_{r,i}/H_{r,j})/\partial\delta_j < 0$ as proven above, there are two possible cases consistent with (A.29):

$$i) \quad \frac{\partial H_{r,j}}{\partial \delta_j} > \frac{\partial H_{r,i}}{\partial \delta_j} > 0 \tag{A.30}$$

$$ii) \quad \frac{\partial H_{r,j}}{\partial \delta_j} > 0 > \frac{\partial H_{r,i}}{\partial \delta_j}. \tag{A.31}$$

In case i), both $H_{r,i}$ and $H_{r,j}$ increases after a positive shock to δ_j. Since $\partial H_r/\partial\delta_j > 0$, equation (A.26) implies $\partial g/\partial\delta_j > 0$: a permanent positive shock to δ_j causes a permanent increase in the common growth rate g. In case ii), $H_{r,i}$ decreases after a positive shock to δ_j, whereas $H_{r,j}$ increases. From the second line in (A.27), g increases; from (A.26), so does H_r, which implies a fall in the number of skilled workers in the manufacturing sectors. □

We now look at the long-term effects of a variation in the productivity of both research sectors through a weakening of the inter-area spillovers in research, i.e. a fall in ψ.

Proposition A.2. *A permanent weakening of the inter-area spillovers in research, ψ, determines a decrease in the growth rate of the system along the balanced growth path, a decrease in the total number of researchers, and a weakening of the previously existing pattern of specialization.*

Proof of Proposition A.2. From the equilibrium condition (15), and since Assumptions 5 and 6 imply $\nu_i > \nu_j$,

$$\frac{\partial\left(H_{r,i}/H_{r,j}\right)}{\partial\psi} = \left(\frac{\delta_i}{\delta_j}\right)^{\frac{1}{1-\eta-\psi_1+\psi_2}} \left(\frac{\nu_i}{\nu_j}\right)^{\frac{\psi}{1-\eta-\psi_1+\psi_2}} \frac{\ln\left(\frac{\nu_i}{\nu_j}\right)}{1-\eta-\psi_1+\psi_2} > 0. \tag{A.32a}$$

Therefore, a decrease in ψ determines a decrease in $H_{r,i}/H_{r,j}$, and thus, given the equilibrium condition in (17), a weakening of the previously existing patterns of specialization.

Along the balanced growth path, the constant growth rate is given by equation (20). Using (9), and depending on which of the two definitions is used, the derivative with respect to ψ is

$$\begin{aligned}\frac{\partial g}{\partial\psi} &= g\left[-\frac{\partial H_{r,i}}{\partial\psi}\frac{1-\eta-\psi_1}{H_{r,i}} + \frac{\partial H_{r,j}}{\partial\psi}\frac{\psi_2}{H_{r,j}} + \ln\left(\nu_i\right) + \frac{\partial H_r}{\partial\psi}\frac{1}{H_r}\right]\\ &= g\left[-\frac{\partial H_{r,j}}{\partial\psi}\frac{1-\eta-\psi_1}{H_{r,j}} + \frac{\partial H_{r,i}}{\partial\psi}\frac{\psi_2}{H_{r,i}} + \ln\left(\nu_j\right) + \frac{\partial H_r}{\partial\psi}\frac{1}{H_r}\right]\end{aligned} \tag{A.33}$$

Substituting the following result obtained from differentiating (18),

$$\frac{\partial H_r}{\partial\psi} = \frac{\left(H-H_r\right)H_r}{H}\frac{\rho}{rg}\frac{\partial g}{\partial\psi}, \tag{A.34}$$

into (A.33), and rearranging, one obtains

$$\begin{aligned}\frac{\partial g}{\partial\psi} &= \frac{rHg}{rH-(H-H_r)\rho}\left[-\frac{1-\eta-\psi_1}{H_{r,i}}\frac{\partial H_{r,i}}{\partial\psi} + \frac{\partial H_{r,j}}{\partial\psi}\frac{\psi_2}{H_{r,j}} + \ln\left(\nu_i\right)\right]\\ &= \frac{rHg}{rH-(H-H_r)\rho}\left[-\frac{1-\eta-\psi_1}{H_{r,j}}\frac{\partial H_{r,j}}{\partial\psi} + \frac{\partial H_{r,i}}{\partial\psi}\frac{\psi_2}{H_{r,i}} + \ln\left(\nu_j\right)\right],\end{aligned} \tag{A.35}$$

which requires an equalisation of the terms inside the brackets. After some manipulations, this accounts to

$$\left(1-\eta-\psi_1+\psi_2\right)\left\{\frac{\partial H_{r,j}}{\partial\psi}H_{r,j}^{-1} - \frac{\partial H_{r,i}}{\partial\psi}H_{r,i}^{-1}\right\} = \ln\left(\nu_j\right) - \ln\left(\nu_i\right). \tag{A.36}$$

The right hand side of (A.36) is negative by Assumptions 5 and 6. As a consequence,

$$\frac{\partial H_{r,j}}{\partial\psi} < \frac{\partial H_{r,i}}{\partial\psi}\frac{H_{r,j}}{H_{r,i}}, \tag{A.37}$$

where the last ratio on the right hand side is lower than one by Assumption 6 and Proposition 2. Since $\partial(H_{r,i}/H_{r,j})/\partial\psi > 0$ as proven above, there are two possible cases consistent with (A.37):

$$i) \quad \frac{\partial H_{r,i}}{\partial \psi} > \frac{\partial H_{r,j}}{\partial \psi} > 0 \tag{A.38}$$

$$ii) \quad \frac{\partial H_{r,i}}{\partial \psi} > 0 > \frac{\partial H_{r,j}}{\partial \psi}. \tag{A.39}$$

In case i), both $H_{r,i}$ and $H_{r,j}$ decrease after a negative shock to ψ, whereas H is constant. Since $\partial H_r/\partial\psi > 0$, equation (A.34) implies $\partial g/\partial\psi > 0$: a permanent negative shock to ψ causes a permanent decrease in the common growth rate g. In case ii), $H_{r,i}$ decreases after a negative shock to ψ, whereas $H_{r,j}$ increases. From the second line in (A.35) and since $H_{r,i}H_{r,j} \times \min(\nu_i, \nu_j) \geq 1$, g decreases; from (A.19), so does H_r. □

Finally, we turn to the implications on the overall level of inequality.

Corollary A2.1. *A permanent reduction in the productivity of an urban area, δ_i or δ_j, or in the inter-area spillovers in research, ψ, decreases the Gini coefficient of the entire system.*

Proof of Corollary A2.1. The Gini coefficient in (23) is clearly decreasing in $w \equiv w_l/w_h = (\alpha/\beta)(H_m/L)$. From Proposition A.1, the derivative of H_m with respect to either δ_i or δ_j is negative, making the derivative of G with respect to either δ positive. From Proposition A.2, $\partial H_m/\partial\psi < 0$ and thus $\partial G/\partial\psi > 0$. □

A.3.1 A Numerical Example

The balanced growth path values resulting from the parametrization in the main text are shown in the first column of Table A.1. In the second column, we summarise the changes following a permanent increase in the productivity of area i's research sector, without any change to the productivity of area j, such that the productivity gap between the two research sectors passes from $\delta_j/\delta_i = 75\%$ to $\delta_j/\delta_i = 50\%$. In the third column, we increase δ_j without changing δ_i, so that $\delta_j/\delta_i = 90\%$

Consistently with Proposition A.1, these shocks have a direct positive effect on the annual growth rate of the economy, but also an indirect positive effect through a reallocation of skilled workers from research to manufacturing. If the shock increases the productivity gap between the research sectors, the economy experiences a radical strengthening of the previously existing patters of specialization, with an increase in the share of researchers employed in area i of 16 percentage points, and a corresponding outflow of unskilled workers. This reallocation stretches the gap in the levels of GDP

Table A.1: Balanced Growth Path Values Under Different Parameters

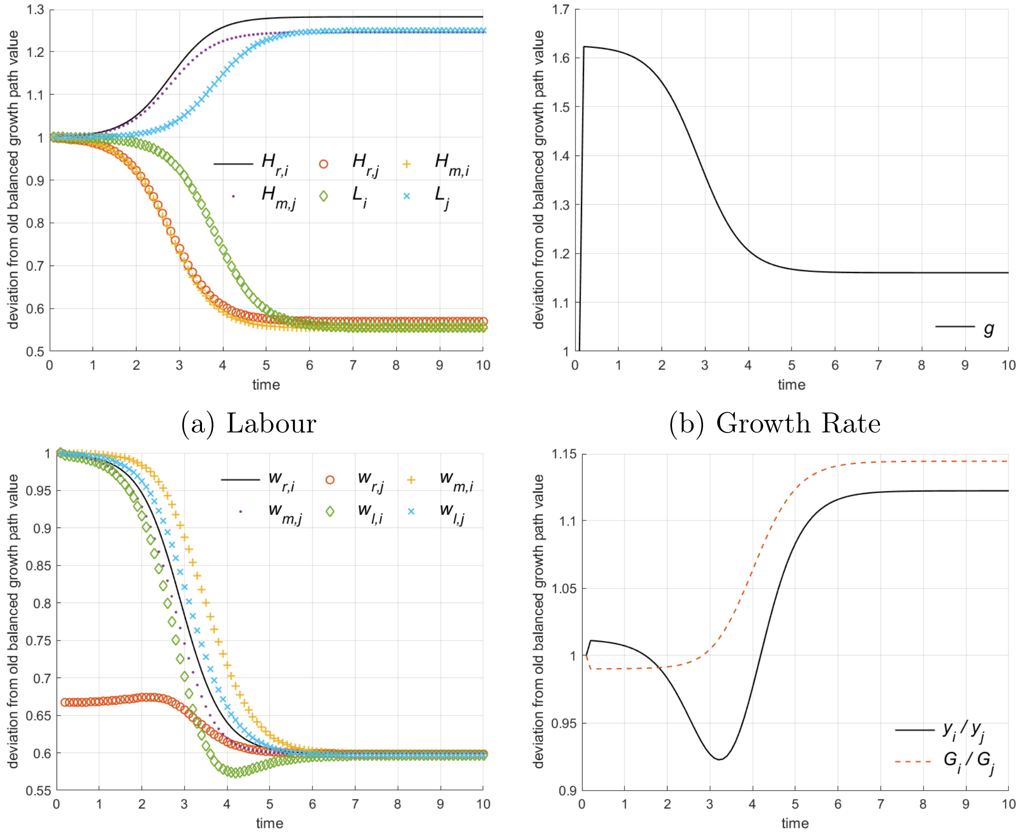
	Baseline $\delta_j/\delta_i = 75\%$	$\uparrow \delta_i$ $\delta_j/\delta_i = 50\%$	$\uparrow \delta_j$ $\delta_j/\delta_i = 90\%$
g	2.00%	2.32%	2.40%
$H_{r,i}/H_{r,j}$	177.78%	400%	110.80%
H_r/H	11.76%	12.06%	12.12%
$H_{r,i}/H_r$	64.00%	80.00%	52.56%
L_i/L	36.00%	20.00%	47.44%
y_i/y_j	106.05%	119.02%	101.04%
G_i/G_j	107.87%	123.42%	101.36%
G	0.416	0.417	0.417

per worker and Gini coefficients; a more heterogeneous composition of the populations in the two areas also leads to an increase in inequality in the overall economy.

Conversely, if the shock increases the productivity of the backward sector, the previously existing patters of specialization are weakened, with a decrease in the share of researchers employed in area i of almost 12 percentage points, and a corresponding inflow of unskilled workers. This reallocation significantly reduces the gap in the levels of GDP per worker and Gini coefficients; however, an increase in the wage gap between skilled and unskilled workers leads to an increase in inequality in the overall economy.

A.3.2 The Transitional Dynamics

The transitional dynamics following the first productivity shock, where δ_i suddenly and permanently increases as in the second column of Table A.1, are given in Figure A.1. The results are similar to the one in the main text but with a greater magnitude, since this shock directly changes the relative productivity of the two areas' research sector. The agglomeration effects are more accentuated, and thus the change in inequality is more pronounced and the cycle more evident.



(a) Labour (b) Growth Rate

(c) Wages (d) Inequalities

Figure A.1: A positive shock to δ_i such that $\Delta\delta_i/\delta_j = +25pp$