Ca' Foscari University of Venice

Department of Economics

Working Paper

Vincenzo Prete
Claudio Zoli

The political economy of educational policies and inequality of opportunity

ISSN: 1827-3580
No. 14/WP/2019

Working Papers
Department of Economics
Ca' Foscari University of Venice
No. 14/WP/2019
ISSN 1827-3580

The political economy of educational policies and inequality of opportunity

Vincenzo Prete
Department of Economics, University of Verona

Claudio Zoli
Department of Economics, University of Verona

Abstract
We propose a political economy model to explain cross-country differences observed in educational policies and to show how such heterogeneity is associated with the level of a country's development and inequality. Parents, heterogeneous in terms of income and their child's ability, vote over the educational policy, by deciding the allocation of a given public budget between basic and higher education. Parents can invest in supplemental private education to increase the probability of their children of being admitted to higher education. When the level of development is low and inequality between social classes is sufficiently large, there is low exchange social mobility in the access to higher education, and educational policies are characterized by a large relative per-student expenditure in higher education.

Keywords
Education, Voting, Development, Inequality, Mobility

JEL Codes
D31, H52, I24, I25

Address for correspondence:
Vincenzo Prete
Department of Economics
University of Verona
Via Cantarane 24
37129 Verona - Italy
E-mail: vincenzo.prete@univr.it

The Working Paper Series
is available only on line
(http://www.unive.it/pag/16882/)
For editorial correspondence, please contact:
wp.dse@unive.it

Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta San Giobbe
30121 Venice Italy
Fax: ++39 041 2349210

The political economy of educational policies and inequality of opportunity*

Vincenzo Prete[†], Claudio Zoli[‡]

May 2019

Abstract

We propose a political economy model to explain cross-country differences observed in educational policies and to show how such heterogeneity is associated with the level of a country's development and inequality. Parents, heterogeneous in terms of income and their child's ability, vote over the educational policy, by deciding the allocation of a given public budget between basic and higher education. Parents can invest in supplemental private education to increase the probability of their children of being admitted to higher education. When the level of development is low and inequality between social classes is sufficiently large, there is low exchange social mobility in the access to higher education, and educational policies are characterized by a large relative per-student expenditure in higher education.

Keywords: Education, Voting, Development, Inequality, Mobility

JEL classification: D31, H52, I24, I25.

*V. Prete gratefully acknowledges the financial support of the "*Guido Cazzavillan Research Fellowship*", that funded this research. He is also grateful to S. Klasen and R. Schwager for their valuable suggestions and comments and to the Department of Development Economics, University of Goettingen, for hospitality.

[†]Department of Economics, University of Verona, Via Cantarane 24, 37129 Verona Italy. E-mail: vincenzo.prete@univr.it.

[‡]Department of Economics, University of Verona, Via Cantarane 24, 37129 Verona Italy. E-mail: claudio.zoli@univr.it.

1 Introduction

Education is usually recognized as one of the main tools for promoting economic growth, redistributing income and achieving equity and social mobility. UNICEF (2015) notes that the main challenge facing the education sector today is to identify and to support policies that improve the efficiency and the equity of education spending. Therefore, given the resource constraints faced by policy-makers and the hierarchical structure of the education system, the design of the educational policies, i.e. the allocation of resources across the various education levels, may have profound consequences on the final outcomes. To this regard, several studies have investigated the impact of the allocation of the education budget on the countries' levels of economic growth and inequality. The main conclusion of this line of research is that the optimal allocation policy, i.e. the education level that should be prioritized, changes according to the country's development level. More specifically, Keller (2010), Patron and Vaillant (2012) and Su (2004) argue that the importance of higher education increases as the country's income level increases. In terms of policy prescriptions, these studies suggest that the optimal allocation rule for a developing country would require to allocate relatively more resources to basic education, while for developed economy the education policy should allocate more resources to higher education[1].

This result is strengthened by empirical literature showing that the returns to education (both private and social) decrease in the level of development and in the level of education, with private returns larger than social ones given the public subsidization of education (Psacharopoulos and Patrinos (2018))[2].

Bearing in mind these results, it becomes surprising to observe the educational

[1] Hidalgo-Hidalgo and Iturbe-Ormaetxe (2012) analyze the effect of policy reforms, changing the allocation of the education budget, in terms of equity and efficiency. Their first finding is that there are no policies achieving at the same time both equity and Pareto efficiency. However, by considering alternative measures of efficiency, expressed in terms of average human capital or productivity, they find that even for developed countries, the policy reform achieving both efficiency and equity would transfer resources from higher to basic education.

[2] Psacharopoulos and Patrinos (2018) find that in low income countries the social returns to education are substantially higher at the primary stage than the secondary and tertiary stages (i.e. 22.1, 18.1 and 13.2). For high income countries the decreasing pattern by education level is maintained, but the differences across education levels are moderate (i.e. 15.8, 10.3 and 9.7). These estimates are also consistent with previous empirical analysis (Psacharopoulos and Patrinos (2004), Brossard and Foko (2006)). Moreover, Gemmel (1996) argued that there exists a key level of human capital that contributes most to growth and that such a level increases as the level of a country's development increases.

policies adopted by the different countries around the world. In particular, as reported by Table 1, there is a remarkable heterogeneity in the allocation policies across regions. The relative per-student expenditure by education level, measured by the ratio between the expenditure on each tertiary student over the per-student expenditure on basic education, ranges from 1.31 to more than 30. OECD economies and countries from East Europe and Central Asia exhibit on average lower ratios of per-student expenditure than African countries. Therefore, reality seems to suggest that African countries tend to allocate their education budget contrary to their development needs (Patron and Vaillant (2012) and Su (2004)). From Table 1, it is also interesting to note that a relatively large per-student expenditure in tertiary education is associated with a higher level of income inequality. In other words, unequal societies, exhibiting high Gini index, tend to adopt education policies more generous towards higher levels of education (Di Gioacchino and Sabani (2009)). In addition, since the access to higher education is unfairly distributed across income groups, it follows that the policies implemented by less developed countries tend to reproduce inequality over time[3]. The unfair outcome of these policies in terms of inequality of opportunity is illustrated by Figure 1, which shows the level of intergenerational educational mobility across countries[4]. Countries with a larger per-student expenditure in tertiary education experience the lowest levels of mobility (see dark blue regions). The regressive aspect of education policies is also discussed by De Fraja (2004) and Fernandez and Rogerson (1995), while UNICEF (2015) reports that in less developed countries education resources are unequally distributed, as 10 per cent of most educated students benefit from almost half of the public education resources. In addition, the inegalitarian nature of education policies in African countries is corroborated by the fact that the households' contribution to higher education is lower than their contribution to basic education, despite the former being more expensive and associated with higher private returns (see Figure 32 in UNICEF (2015)).

This astonishing scenario has stimulated researchers to understand how education policies, favoring only a small elite, can be implemented. The explanation proposed

[3]UNICEF (2015) reports that the allocation policies in African countries favor children from wealthier families. A fraction ranging from 60 to 97 per cent of students enrolled at the tertiary level of education, indeed, comes from the wealthiest quintile.

[4]The level of relative education mobility is defined as the coefficient from a regression of respondents' years of schooling on the highest years of schooling of their parents. That is, this coefficient measures the effect of one additional year of schooling of parents on their children's years of schooling.

by this line of research is that in less developed countries an elite holds the political power and may decide its preferred redistributive policy. Gradstein (2003) shows that educational policies unfavorable to the poor arise in contexts where income inequality is high and the access to higher education is determined by rent seeking. Su (2006) proposes a political economy model where the elite has to decide the allocation of a given education budget across two levels, i.e. basic and higher. The complementarity across the two education levels gives the incentive to the elite to allocate more resources to higher education at the expense of basic education, in order to reduce the possibility of the poor to receive higher education[5]. Naito and Nishida (2017) extend the study by Su (2006) by proposing a model where both the size of the education budget and its allocation between basic and higher education is decided via majority voting. Their results show that the optimal education policy depends on the level of human capital of the median voter. When this level is lower than a given threshold, the tax rate, i.e. the education budget, is low and all resources are allocated to basic education. If the low value of human capital of the median voter is associated with large inequality and underdevelopment, the implication of the model is that inequality prevents growth and underdevelopment feeds itself. Different than Su (2006), Naito and Nishida (2017) do not assume that the rich have the incentive to exclude the poor from higher education and focus on the effect of private cost of higher education.

Other studies show that the choice to implement a more egalitarian education policy where rich people subsidize the education of the poor is associated with: the threat of revolution or predatory behavior by the poor (Acemoglu and Robinson (2000) and Grossman and Kim (2003)), the positive externalities of education (Bourguignon and Verdier (2000)).

This paper proposes a political economy model to explain how the country's levels of development and inequality affect the allocation rule of the education budget. In addition, we try to understand how this rule changes over time. By comparing the two panels of Table 1, indeed, it is possible to note that the relative per student

[5] Di Gioacchino and Sabani (2009) propose a model to explain the relationship between educational policies and the level of wealth concentration among OECD countries. In their model they consider different education levels as alternative public investments with contrasting effect on the future labor income distribution. The main result is that countries where wealth concentration is larger than income inequality, tend to allocate more resources to high education at the expense of basic education.

expenditure in African countries decreased over the period 2008-17, although it remained higher compared with the other regions. These lower ratios can be explained with a less elitist higher education, as illustrated by Figure 2, indeed, enrollment in tertiary education exhibits an increasing pattern over the last decade. This trend can lead to a lower per tertiary student expenditure and seems to support the idea that the elite decides to allow access to higher education to children from other social classes.

By studying how the level of initial development and inequality affect the educational policy and by analyzing the dynamics over time, this paper aims to offer a political economy explanation to this observed evidence. We consider an economy where each individual lives for two periods: childhood and parenthood. Children attend school and do not take any economic or political decisions, while parents work, earn an income and are called to vote to decide the educational policy, i.e. how to allocate a given public budget across the different education levels. More specifically, we assume a two-stage hierarchical education system (basic and higher education), where the level and the quality, which is measured by the per-student expenditure, of the education received during childhood, determine the future income of individuals when adult. The two education levels are associated with two income classes, rich and poor, and the inequality among them depends on the allocation rule. We assume that only a limited share of the children's generation has access to higher education and that the access is based on the performance of an admission test. This performance reflects the child ability, which is given by the sum of two components: the child innate talent and the effect of the family environment. That is, the child ability is the combination of nature and nurture. However, parents can decide to invest in supplemental private education in order to improve their children's performance and increase their probability to receive higher education.

The allocation of a given education budget is determined by rich parents through majority voting. With this assumption we capture the idea that in less developed countries there is an elite holding the political power. Here, the elite consists of high-educated parents, who decide how to allocate a given education budget. Different than Su (2006), here we introduce social exchange mobility among the two social classes. In other words, we consider a sort of race for access to higher education, where child's ability still plays a crucial role. In particular, even if children from rich families have more opportunity to access higher education than their peers from poor backgrounds, because of the presence of nurture effect and the possibility to invest more resources in supplemental education, there are always high talented children from poor families who may benefit from higher education. That is, rich parents cannot always compensate with private education their children's low level of talent.

The *main result* of the model is that: when the country's level of development is low and income inequality is large the allocation rule tends to prioritize per-student investments in higher education and the level of exchange mobility is low, as more than half of children admitted to high education are from rich families. This policy leads to higher income inequality which in turn implies higher inequality of opportunity in terms of access to higher education. However, this trend cannot be sustained over a long period. When the income of the poor starts to increase, the rich lose their income advantage due to investments in private education and the inequality of opportunity in the access to higher education declines, given that the share of rich children admitted to higher education declines. Therefore, a majority supporting investment in higher education will continue to exist only if the rich decide to increase the share of children admitted to higher education. These results seem to be consistent with the observed empirical evidence of higher enrollment rates (see Figure 2), while the relationship between inequality of opportunity and development is confirmed by Brunori et al. (2013), who find an inverted-U shaped relationship between the level of development and inequality of opportunity.

The rest of the paper is organized as follows: Section 2 introduces the environment of the model. Section 3 presents the "static" majority voting equilibrium under different scenarios characterized by different levels of initial income inequality. Section 4 analyzes the impact of the educational policy on social mobility and discusses the dynamics of the model. Section 5 concludes.

Table 1: Government expenditure per student on a given level of education (share of GDP per capita).[6]

Panel A: Relative government expenditure across education levels (years 2000 - 2007)

	OECD		East Europe Central Asia		Middle East North Africa		Sub Saharan Africa		South Asia		East Asia Pacific		Latin Amer.[7]	
	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini
Obs.	176	135	18	87	16	16	65	50	8	11	13	16	52	103
Mean	1.64	32.83	1.31	33.26	3.40	36.77	31.60	44.03	6.83	36.17	2.19	38.19	2.27	52.52
St.dev	0.66	6.16	0.43	5.12	1.38	4.35	42.81	7.54	3.03	4.70	2.49	5.09	1.05	3.96
Min	0.42	24.40	0.67	16.20	1.46	28.60	2.15	29.80	3.58	30.40	1.06	27.80	0.99	38.10
Max	3.49	52.80	2.23	42.30	6.02	44.80	232.46	64.80	13.70	43.80	10.39	46.10	5.19	61.60

Panel B: Relative government expenditure across education levels (years 2008 - 2017)

	OECD		East Europe Central Asia		Middle East North Africa		Sub Saharan Africa		South Asia		East Asia Pacific		Latin Amer.	
	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini	$\frac{\text{Tert.}}{\text{Prim.}}$	Gini
Obs.	175	215	42	119	12	19	119	56	35	13	38	28	57	120
Mean	1.40	32.03	1.78	32.28	1.98	34.08	17.21	44.16	5.65	34.53	1.63	37.57	1.74	47.47
St.dev	0.52	5.12	5.03	4.74	1.05	3.71	31.92	8.69	3.89	3.47	0.96	3.91	0.90	4.32
Min	0.42	23.70	0.44	24	1.25	27.60	0.76	31.50	1.95	29.80	0.18	28.70	0.68	36.70
Max	3.01	49	33.54	42.8	4.92	42.10	288.65	63.40	20.69	39.80	4.78	45.50	5.85	56.20

Total general (local, regional and central) government expenditure (current and capital) on a given level of education (primary, tertiary, etc) minus international transfers to government for education, divided by the number of student enrolled at that level of education. This is then expressed as a share of GDP per capita. Authors' elaboration from UNESCO Institute for Statistics data.

Chile is listed as OECD country.

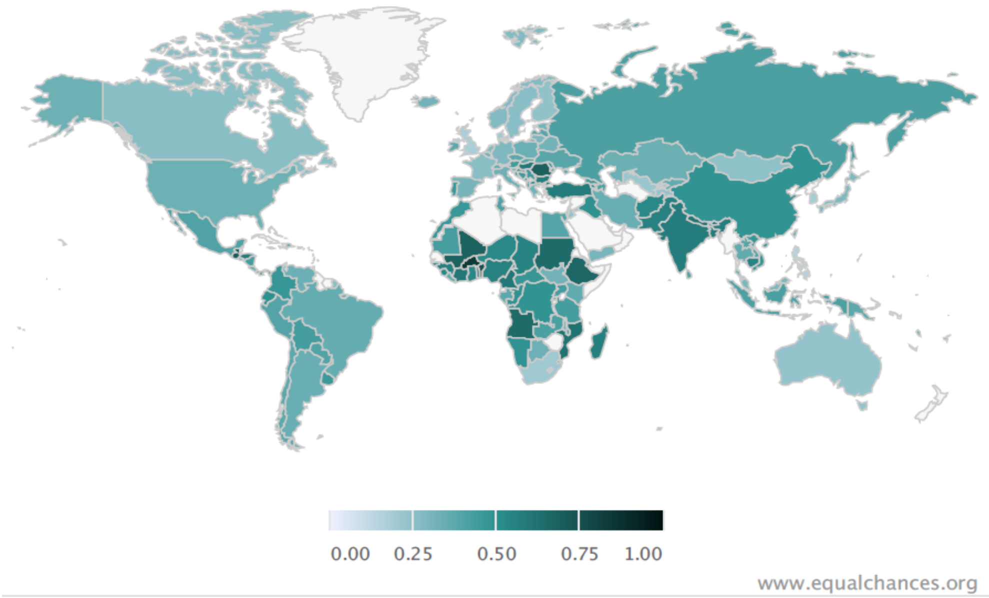

Fig.1: Relative education mobility

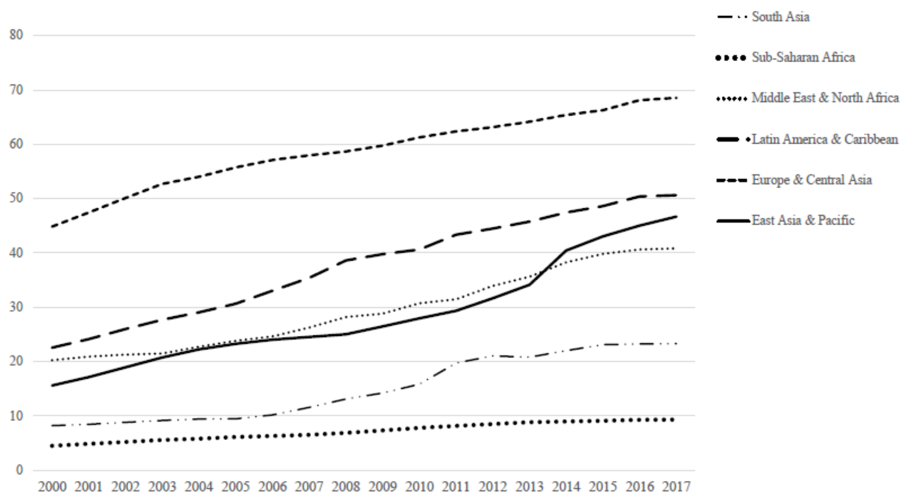

Fig. 2: Gross enrollment ratio in tertiary educ. (Authors'elaboration from World Bank).

2 The model

2.1 The economic environment

This paper considers an economy where individuals live for two periods: childhood and parenthood. In each period there are two generations, children and parents, which we assume to be equally sized and constant over time. Each parent has only one child and to simplify the exposition we normalize to one the size of both generation.

During childhood, individuals receive education and do not take any economic or political decisions. After completing education, children become adults (parents) and work, by earning a labor income that depends on the level and the quality of the education received during childhood, which in turn depend respectively on their level of ability and the education policy implemented.

We assume that there are two income classes denoted by y_r and y_p respectively, with $y_r > y_p$, which reflect the distinction between high and low-educated individuals[8].

In addition to income, there is a second source of heterogeneity across parents: the ability level of their child. The ability of each child is given by the sum of two components. The first component is the innate talent, denoted by ω, which we assume to be uniformly distributed over the support $(0, 1)$. The second component, labeled by λ, deals with the family environment and captures the idea that high-educated parents bequeath part of their human capital to their children. Thus, the ability of each child can be interpreted as the combination of *nature* and *nurture*. The former randomly assigns to each child a rank within the ability distribution regardless of their parent background, while the latter introduces a gap between the ability distributions of rich and poor, by increasing the talent of each rich child by an amount λ, that can be interpreted as a measure of the correlation between the innate talent and the parental income. When $\lambda = 0$ abilities are completely chosen by the nature and nurture has no say in this aspect.

As we will see in the paper, the ability level of each child is crucial in determining the level of education that she receives and hence her future income.

[8] In the remainder of the paper we equivalently refer to rich (poor) parents as high-educated (low-educated) or skilled (unskilled) individuals respectively.

2.2 The education system

We consider a hierarchical education system with two subsequent levels: basic and higher education. These two levels can be interpreted as compulsory school and university. *Basic education* is universal and provides all children with the same basic level of skills. After completing this first level, children may advance to the next education level, by accessing to *higher education*. However, in contrast with the basic education, the access to higher education is more selective and restricted to a limited fraction (θ) of children. This fraction captures the idea that there are some constraints that prevent policy-makers from expanding the access to higher education without cost. Therefore, given that only some children may benefit from advanced education, the challenge for the policy-maker is to make the access to higher education as equal as possible, by ensuring that high-ability children receive advanced education regardless of their parental income class.

We assume that children admitted to higher education are identified according to the performance achieved in an admission test. Therefore, the fraction θ represents the share of children with the highest test performance. This performance is the sum of two components, namely the level of child's ability and a possible additional investment in supplemental private education (e). By resorting to supplemental private education (*top-up*) parents may increase the test performance of their children and then their chances to be admitted to higher education. While public (basic and higher) education is entirely financed by a proportional income tax, private education is funded only by parents who use it, by paying a per unit cost that we normalize to one.

Basic education provides all children with the same standard set of skills, but some parents may decide to supplement the public provision with an additional investment that boosts the test mark of their child, thereby increasing her probability of being admitted to higher education. It will follow that for the same ability level, children from rich families, have a larger probability to access to higher education than children from poor families having fewer resources to invest in private education, and a higher marginal utility of income.

2.3 Education policy, income classes and inequality

In each period t, the policy-maker levies a proportional income tax τ collecting an amount of revenue $R^t = \tau^t \mu^t$, where the term μ^t denotes the average income of the economy[9], which is defined as $\mu^t = \theta y_r^t + (1-\theta)\, y_p^t$. Tax revenues finances education policies and this budget is allocated across the two education levels.

Let α denote the fraction of tax revenues allocated to higher education, then the per-student expenditure (g_B) in basic education and (g_U) in higher education are

$$g_B^t = \left(1-\alpha^t\right) R^t \tag{1}$$

$$g_U^t = \frac{\alpha^t R^t}{\theta}. \tag{2}$$

After completing education children become adults and enter the labor market, where they earn an income which depends both on the level and the quality of the received education[10].

Let $y_0 > 0$ be the level of a minimum income that individuals receive regardless of their education background. Then, the labor income of low educated individuals, who received only basic education, is formalized by

$$y_p^{t+1} = y_0 \left(1 + k_1 g_B^t\right), \tag{3}$$

where $k_1 > 0$ is a scale parameter. Note that, when the education budget is entirely allocated to higher education, then the quality of basic education is extremely low and unskilled individuals earn the minimum income y_0.

High-educated individuals, who have benefitted from higher education, earn instead the following labor income

$$y_r^{t+1} = y_0 \left(1 + k_1 g_B^t\right) \times \left(1 + k_2 g_U^t\right), \tag{4}$$

where $k_2 \geq k_1 > 0$. That is, the income of high-educated individuals corresponds to the income of the low-educated ones increased by an additional component that depends on the quality of the higher education. Given the definitions of the per-student expenditure in basic and higher education in (1) and (2), the two income

[9] Here we consider the computation of the average income only for the adult population.

[10] In this model the quality of a specific education level is assumed to be measured by the amount of per-student expenditure in that level.

levels earned by unskilled and skilled individuals can be rewritten respectively as

$$y_p^{t+1} = y_0 \times \left(1 + k_1 (1-\alpha) R^t\right), \quad (5)$$

$$y_r^{t+1} = y_0 \times \left(1 + k_1 (1-\alpha) R^t\right) \left(1 + \frac{k_2 \alpha R^t}{\theta}\right). \quad (6)$$

Unskilled children represent the poor (p) class, whose income is negatively correlated with the share of the public budget allocated to higher education. Thus, public investments in basic education represent a sort of future income insurance and can be interpreted as a "pro-poor" redistribution, as they reduce the income inequality between the two social classes. On the other hand, more generous investments in higher education tend to worsen the quality of basic education and to increase the level of future income inequality.

Figure 3 illustrates the relationship between the allocation rule and income inequality which is expressed in terms of the Lorenz curve. The higher the share of education budget allocated to higher education, the larger is the future income inequality. Moreover, for a given education budget and an allocation rule, the level of income inequality decreases when the access to higher education is less elitist[11].

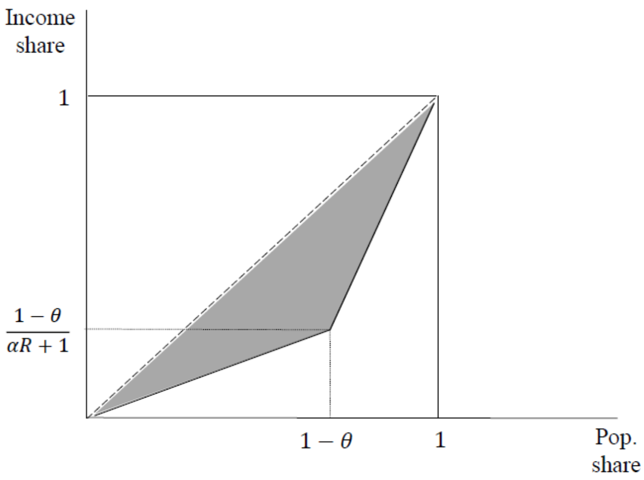

Fig. 3: Education policy and income inequality
(Lorenz Curve).

[11]The Gini index G is double the area between the Lorenz curve and the line of perfect equality (dark area), its value is $G = (1-\theta)\frac{\alpha R}{1+\alpha R}$.

2.4 The timing

The timing of the model is the following: parents, taking the size of the education budget as exogenous, first vote to decide the allocation rule α. Then, they decide whether or not and how much to invest in private education in order to increase their child's probability to access higher education. *Child's ability is known before the vote. Moreover, we assume that only rich (high-educated) parents vote.* This last assumption captures the idea that in less developed countries there is an elite that holds the political power. In this model the elite consists of the rich (high-educated) parents.

To solve the model we proceed by backward induction, starting from the analysis of parents' preferences and their decision to invest in private education for a given educational policy. Then, we derive the optimal allocation rule for rich parents who resort or not to private education.

2.5 Parents' preferences and decisions

Parents care about their consumption and their child's future labor income. More specifically, consumption is funded by the available resources which are given by the labor income net of the taxation, i.e. $x_i = y_i (1 - \tau)$, minus the investment in supplemental private education $e_{i,j}$, where $i = p, r$ denotes the income class of the parent, while the subscript j indicates that the amount of resources invested in supplemental education depends on the child's characteristics, such as his level of innate talent. The child's future labor income, which depends on the level and the quality of the education received, instead, is defined by (5) or (6) for low-educated and high-educated children respectively.

The utility of the generic parent belonging to the income class i, whose child may be part of a different income class i', can be written as

$$u\left(x_i, y_{i'}^{t+1}\right) = v\left(x_i\right) + \rho y_{i'}^{t+1}, \tag{7}$$

where the parameter ρ measures the degree of parental altruism towards the child's income, while the function $v(\cdot)$, which quantifies the utility impact of the current level of net income x, is defined as follows

$$v(x) := \begin{cases} ax & \text{if} \quad x < \overline{x} \\ a\overline{x} + b(x - \overline{x}) & \text{if} \quad x \geq \overline{x} \end{cases} . \tag{8}$$

Function $v(\cdot)$ represents the most simple way to formalize the effect of decreasing marginal utility of income. More specifically, the term $\overline{x}$ denotes a threshold that can be interpreted as a sort of "poverty line" in the distribution of net income. The parameters a and b, instead, represent the marginal utility of net income of parents enjoying an amount of resources respectively lower or higher than the threshold $\overline{x}$, with $a > 1 > b$.

For a given educational policy, i.e. the allocation rule α, the generic parent belonging to the income class i, decides to invest in supplemental education, thereby having a high-educated child, if the utility with this investment is at least larger than the utility achieved without and having a low-educated child. This condition can be formalized as

$$v(x_i - e_i) + \rho y_r^{t+1} \geq v(x_i) - \rho y_p^{t+1}. \tag{9}$$

By replacing the definitions (5) and (6) and after some manipulations, condition (9) becomes

$$Z \geq v(x_i) - v(x_i - e_i), \tag{10}$$

where the term Z is

$$Z := \rho \left(y_r^{t+1} - y_p^{t+1} \right) = \rho y_0 \left(1 + k_1 (1 - \alpha) R\right) k_2 \frac{\alpha R}{\theta}, \tag{11}$$

which is a measure of the discounted future income advantage of having a high-educated child. The income differential between high and low-educated children is given by the minimum guaranteed income (y_0), weighted by the degree of parental altruism (ρ), the labor income of low-educated children $\left(y_p^{t+1} = (1 + k_1 (1 - \alpha) R)\right)$ and the quality of higher education $\left(g_U = \frac{\alpha R}{\theta}\right)$. The level of e_i solving condition (10) with equality, represents the maximal investment in top-up that the generic parent belonging to the income class i is willing to afford to have a high-educated child. The maximal top-up, denoted by $\widehat{e}_i$, will be a crucial ingredient of the next section which discusses the access to higher education for children from different backgrounds.

We analyze how the maximal investment in private education changes with the level of parental income. To this regard, there are three alternative scenarios that may occur when the generic parent decides to afford the maximal investment in top-up: *Case A,* the disposable income after such investment is still larger than the threshold $\overline{x}$; *Case B,* the disposable income after the maximal top-up becomes lower than the threshold $\overline{x}$; *Case C,* the disposable income before the maximal top-up is already lower than the threshold $\overline{x}$. Each of these three cases is associated with a specific formulation of the function $v(\cdot)$, with the maximal top-up $\widehat{e}_i$ which is obtained by replacing this specification, given by (8), into condition (10). More specifically, the maximal top-up associated with *Case A* solves the following equation

$$Z = a\overline{x} + b(x_i - \overline{x}) - a\overline{x} - b(x_i - \overline{x} - e_i), \tag{12}$$

which implies that

$$\widehat{e}_i = \frac{Z}{b}. \tag{13}$$

The maximal top-up corresponding to *Case B* is obtained from the following condition

$$Z = a\overline{x} + b(x_i - \overline{x}) - a(x_i - e_i), \tag{14}$$

with

$$\widehat{e}_i = \frac{Z}{a} + \frac{(a-b)}{a}(x_i - \overline{x}), \tag{15}$$

while the maximal investment in private education associated with *Case C* is given by

$$Z = a(x_i) - a(x_i - e_i), \tag{16}$$

with

$$\widehat{e}_i = \frac{Z}{a}. \tag{17}$$

In all cases the maximal top-up increases the larger is the future income advantage of having a high-educated child, that is the larger is the term Z. Given (11), it follows that for a given allocation rule α the maximal top-up is positively associated with the size of the education budget R, the degree of parental altruism ρ, the level of the minimum income y_0 and the parameters k_1 and k_2.The less elitist is the access to higher education, the larger is θ and the lower is the maximal investment in top-up.

This is the case because when the share of children admitted to higher education is large, the income advantage of receiving higher education decreases, as the quality of higher education, measured by the level of per-student expenditure, decreases. By deriving (11) with respect to α, one obtains that the maximal top-up and α are positively associated if $\alpha < \overline{\alpha} = \frac{1}{2}\left(1 + \frac{1}{k_1 R}\right)$. Given the complementarity between the two education levels as in (6), a minimum level of the quality for the basic education has to be guaranteed in order to benefit from higher education. Therefore, educational policies particularly generous towards higher education (with $\alpha > \overline{\alpha}$), by worsening the quality of basic education, tend to reduce the incentive of parents to invest in private education, as the benefit of higher education declines.

Lastly, for a given level of Z, a lower marginal utility of net income, measured either by the term a or b tends to increase the maximal willingness of parents to invest in supplemental private education. In all the three cases analyzed the maximal top-up is independent from the child's ability level, with all parents belonging to the same income class who are willing to afford the same maximal investment in private education. The intuition for this result is due to the fact that the level of child's ability impacts only on the chance to be admitted to higher education, but it does not affect the benefit of being high-educated, which is always described by the income function (6).

2.6 The access to higher education and the role of initial income inequality

As anticipated in Section 2.2, the access to higher education is restricted to a limited share θ of children, which is identified according to the performance of an admission test. There is no uncertainty about the determination of the test performance s, which can be formalized, for the generic child j whose parent belongs to the income class i, as follows

$$s_{j,i} = \omega_{j,i} + \lambda_i + \beta e_{j,i}. \tag{18}$$

The term ω represents the child's level of innate talent, which is assumed to be uniformly distributed over the support $[0, 1]$,with the associated *cdf* given by $F(\omega) = \omega$.

The parameter λ, instead, measures the nurture effect due to family environment that can be interpreted as a "bonus" that increases the innate talent of children from rich families, that is $\lambda_r > \lambda_p = 0$.

Moreover, parents may improve the performance of their child, by investing e in supplemental education. The marginal impact of this investment on the child's performance is measured by the parameter $\beta > 0$. Therefore, for a given level of innate talent, children from a rich family achieve a larger test score than their peers from a poor background, because of the nurture effect and the possibility to benefit from larger investments in supplemental education.

Given the limited number of positions available for higher education, the policy-maker needs to identify a threshold score s for the admission test, such that only children achieving at least that level are admitted to higher education. We assume that the policy maker decides that the minimum score required to be admitted to higher education is s_0. Then, each parent, either rich or poor, taking into account s_0 and his child's talent, decides whether or not to invest in supplemental private education in order to reach the admission score, thereby allowing his child accessing higher education. The following Lemma summarizes the decision about the investment in top-up of rich and poor parents.

Lemma 1 *Given the admission threshold s_0, the parent belonging to the income class i, with $i = p, r$, and whose child ability is $\omega_{j,i} + \lambda_i$, with $\lambda_r > \lambda_p = 0$, decides the following investments in supplemental education:*

i) $e_{j,i} = 0$ *if* $\omega_{j,i} + \lambda_i \geq s_0$;

ii) $e_{j,i} = \frac{s_0 - (\omega_{j,i} + \lambda_i)}{\beta}$ *if* $s_0 - \widehat{e}_i \leq \omega_{j,i} + \lambda_i < s_0$;

iii) $e_{j,i} = 0$ *if* $\omega_{j,i} + \lambda_i < s_0 - \widehat{e}_i$.

That is, the generic parent i does not invest in supplemental private education either when his child's ability is larger than the admission threshold s_0 (Lemma 1, *case (i)*), or when his child does not achieve the admission score even with the maximal top-up $\widehat{e}_i$ (Lemma 1, *case (iii)*). In both cases, the investment in supplemental education is wasteful. When the child's ability level is instead intermediate, that is $\omega_{j,i} + \lambda_i \in [s_0 - \widehat{e}_i; s_0)$, (Lemma 1, *case (ii)*) the parent decides to invest in private education, by purchasing precisely the amount needed to reach the admission score s_0.

Recall that children from a rich family benefit from the nurture effect, that increases their level of innate talent ($\lambda_r > \lambda_p = 0$). In addition, rich parents may have a lower marginal utility of net income and then a larger willingness to invest in top-up than poor parents ($\widehat{e}_r \geq \widehat{e}_p$). Therefore, given the parents' decisions to invest in supplemental education, described by Lemma 1, it follows that there are two different distributions of the admission test performance for children from different backgrounds.

Given the definition of the test score in (18), the distribution of the test performance corresponds to the distribution of child's talent ω for all children who do not receive private education (i.e. children with talent $\omega_{j,i} < s_0 - \widehat{e}_i - \lambda_i$ and $\omega_{j,i} \geq s_0 - \lambda_i$), while there is a bunching at the level of the admission threshold s_0 for all children who resort to private education (i.e. children with talent $\omega_{j,i} \in [s_0 - \widehat{e}_i - \lambda_i; s_0]$).

The admission score s_0, chosen by the policy maker, has to guarantee that in equilibrium the share of children admitted to higher education corresponds to the limited number of positions available θ.

Given that the investment in supplemental education is aimed at achieving the admission threshold s_0, by solving (18) for $\omega_{j,i}$ one obtains the talent level of the marginal child accessing to higher education with the maximal top-up. This level can be written as

$$\widehat{\omega}_r = s_0 - \lambda_r - \beta \widehat{e}_r \tag{19}$$

for a child from a rich family and as

$$\widehat{\omega}_p = s_0 - \beta \widehat{e}_p, \tag{20}$$

for a child with a poor (low-educated) parent, because parents whose child's talent is lower than the threshold $\widehat{\omega}_i$, with $i = p, r$, do not invest in supplemental education as their child cannot reach the admission threshold s_0 even with the maximal top-up. Given that the share of children coming from a rich family is θ, the following two terms $\theta \times F(\widehat{\omega}_r)$ and $(1-\theta) \times F(\widehat{\omega}_p)$ denote the fraction of children from rich and poor background respectively, who cannot achieve the admission score s_0, thereby receiving only basic education. In equilibrium, the sum of these two shares has to be equal to $1 - \theta$, since only a fraction θ of children benefits from higher education. This consistency condition is formalized as

$$\theta F\left(\widehat{\omega}_r\right)+(1-\theta) F\left(\widehat{\omega}_p\right)=1-\theta. \tag{21}$$

By using the definitions (19) and (20), the above condition can be rewritten as

$$\theta F\left(s_0-\beta \widehat{e}_r-\lambda_r\right)+(1-\theta) F\left(s_0-\beta \widehat{e}_p\right)=1-\theta. \tag{22}$$

The level s_0 ensuring that in equilibrium exactly a share θ of children is admitted to higher education is obtained from condition (22). The next proposition formalizes how the policy maker, taking into account parents' choices, decides the equilibrium admission score.

Proposition 1 *When the access to higher education depends on the performance of an admission test and parents may improve their child's performance by investing in supplemental education, as described by Lemma 1, the admission score such that in equilibrium only a share θ of children receives higher education, is*

$$s=(1-\theta)+\theta\left(\beta\left(\widehat{e}_r-\widehat{e}_p\right)+\lambda_r\right)+\beta \widehat{e}_p. \tag{23}$$

That is, the "equilibrium" admission score is positively associated with the maximal investments in supplemental education of both rich and poor parents. If parents invest more resources in supplemental education the children's performances improve, thus the policy maker has to increases the admission threshold in order to still admit only a share θ of children. In addition, the equilibrium threshold is increasing in the parameters β and λ_r, which measure respectively the impact of top-up on the performance and the nurture effect for children from rich families. Lastly, the more elitist is the access to higher education, i.e. the lower is θ, the higher has to be the admission threshold s. This condition is crucial to ensure that for each level of $\theta \in (0,1)$ there exists only an admission score s satisfying condition (23). By using (23), one may obtain from (19) and (20) the talent level of the marginal child who is admitted to higher education with the maximal top-up, which is formalized by

$$\widehat{\omega}_r=(1-\theta)\left(1-\lambda_r\right)-(1-\theta) \beta\left(\widehat{e}_r-\widehat{e}_p\right), \tag{24}$$

for a child from a rich family, and by

$$\widehat{\omega}_p=(1-\theta)\left(1-\lambda_r\right)+\lambda_r+\theta \beta\left(\widehat{e}_r-\widehat{e}_p\right). \tag{25}$$

for a child from a poor background. The threshold $\widehat{\omega}_i$ defined by (24) and (25) represents the minimum level of talent such that the investment in supplemental private education is profitable. The nurture effect λ_r and the different maximal willingness to invest in private education of parents from a different income class $(\widehat{e}_r - \widehat{e}_p)$ imply that for a child from a rich family is easier to reach the admission threshold, than a child from a poor background, i.e. $\widehat{\omega}_r < \widehat{\omega}_p$. The next Remark summarizes this result.

Remark 1 *The initial income inequality impacts on inequality of opportunity and education mobility through two channels: the different maximal investment in private education (i.e., $\widehat{e}_r > \widehat{e}_p$) and the nurture effect (i.e., $\lambda_r > \lambda_p = 0$). Therefore, the last rich child admitted to high education is less talented than the last poor child admitted, i.e. $\widehat{\omega}_r < \widehat{\omega}_p$.*

Finally, by subtracting the nurture effect λ_r from (23), one obtains the level of the innate talent of the rich child achieving the required admission score s with the lowest top-up. This level can be written as

$$\widetilde{\omega}_r = (1-\theta)(1-\lambda_r) + \beta(\theta\widehat{e}_r + (1-\theta)\widehat{e}_p). \tag{26}$$

Children with a level of talent higher than $\widetilde{\omega}_r$ do not need private education, as their ability (i.e., the sum of innate talent and nurture effect) is greater than the required admission threshold s. Recall that for children from poor families the threshold $\widetilde{\omega}_p$ corresponds to the required admission score s, as ability and talent coincide given that $\lambda_p = 0$.

The admission score s and the talent thresholds $\widehat{\omega}_r$ and $\widehat{\omega}_p$ are crucial to analyze the level of social exchange mobility. These three elements, defined respectively by (23), (24) and (25), are influenced by the level of the maximal investment in private education $\widehat{e}_i$ of rich and poor parents, which in turn depends on the level of their net income, as described in Section 2.5. Therefore, different levels of the initial income inequality between rich and poor parents, changing the rank of their net income with respect to the threshold $\overline{x}$, have an impact on the maximal willingness to invest in private education and, hence, on the access to higher education for children from different backgrounds. We consider three alternative scenarios, denoted respectively as high, medium and low initial income inequality. The next subsections describe the access to higher education for each of these three cases.

2.6.1 High income inequality

The case of *high initial inequality* is such that rich (poor) parents have a disposable income post (pre) the top-up investment greater (lower) than the threshold $\overline{x}$. Under this scenario, the maximal investment in supplemental education of rich and poor parents is described by (13) and (17) respectively. Then, by replacing these two levels within (23), the equilibrium admission score, ensuring that a share θ of children accesses to higher education, is

$$s^h = (1-\theta) + \beta Z \left(\frac{\theta}{b} + \frac{(1-\theta)}{a} \right) + \theta \lambda_r, \tag{27}$$

where the superscript h stands for "high inequality". From (24) and (25), we have that the marginal child accessing to higher education with the maximal top-up has the following level of talent

$$\widehat{\omega}_r^h = (1-\theta)(1-\lambda_r) - (1-\theta)\beta \frac{Z}{b}\left(\frac{a-b}{a} \right) \tag{28}$$

if her parent is rich (high-educated), and

$$\widehat{\omega}_p^h = (1-\theta) + \theta\beta \frac{Z}{b}\left(\frac{a-b}{a} \right) + \theta\lambda_r \tag{29}$$

if she comes from a poor family. Finally, by replacing (13) and (17) into (26) one obtains that the talent level of this marginal child admitted to higher education with the lowest top-up is

$$\widetilde{\omega}_r^h = (1-\theta)(1-\lambda_r) + \beta Z \left(\frac{\theta}{b} + \frac{(1-\theta)}{a} \right). \tag{30}$$

By using the two thresholds $\widehat{\omega}_r^h$ and $\widetilde{\omega}_r^h$ one may partition the group of voters, i.e. the population of rich parents, into different subgroups according to the child level of talent. Recall that, given the complementarity of the two education levels, the income advantage of having a highly educated child formalized by Z in equation (11) is maximized when $\alpha = \overline{\alpha} = \frac{1}{2}\left(1 + \frac{1}{k_1 R}\right)$. Then, by assuming that $\overline{\alpha} < 1$, i.e. $R > \frac{1}{k_1}$, the two thresholds $\widehat{\omega}_r^h$ and $\widetilde{\omega}_r^h$ appears as illustrated in Figure 4.

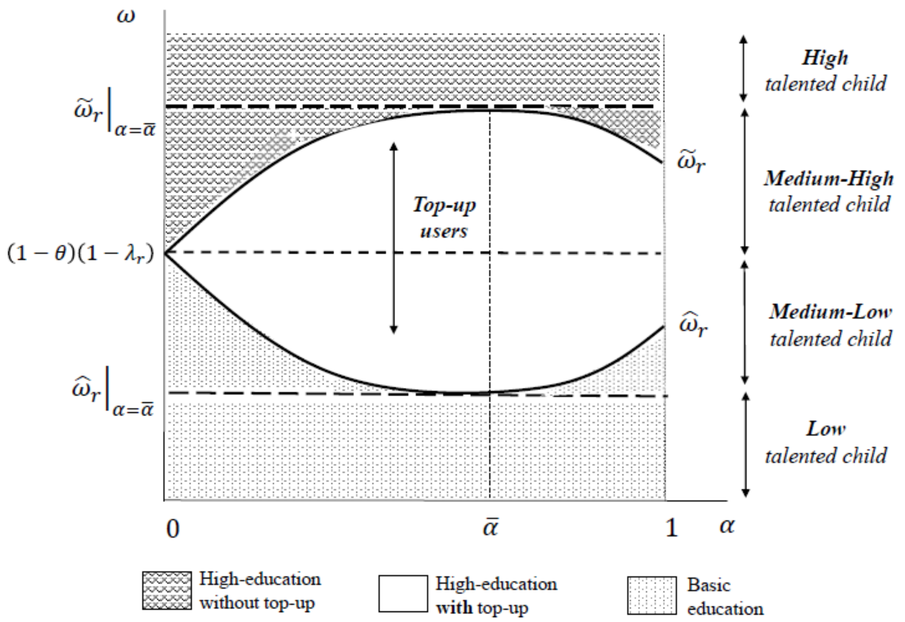

Fig. 4: Access to higher education with high inequality.

For each allocation rule $\alpha > 0$ (reported on the horizontal axis) there are four different groups of voters (rich parents) identified according to the level of their child's talent reported on the vertical axis.

More specifically, there are two extreme groups, labeled as *high-talented* and *low-talented* children, that include respectively children who are always admitted to higher education without top-up and children who always receive only basic education. Between these two extreme groups, there are two intermediate groups of children, i.e. *medium-high-talented* and *medium-low-talented*, consisting of children who need supplemental education to enter higher education.

To identify the two extreme groups we need to define the maximal income advantage of having a high-educated child. Then, by replacing this value within the thresholds (30) and (24), we obtain the two levels of talent that identify the marginal child belonging to each of these two group. Let $\overline{Z}$ denote the maximal income advantage of having a highly-educated child, which is obtained by replacing $\alpha = \overline{\alpha}$ into

(11), that is

$$\overline{Z} = \frac{\rho y_0}{4\theta}\left(1+k_1R\right)^2\frac{k_2}{k_1}.$$

The group of *high-talented* children, who are always admitted to higher education without top-up, includes all children whose talent is greater than the following threshold

$$\left.\widetilde{\omega}_r^h\right|_{\alpha=\overline{\alpha}} = (1-\theta)(1-\lambda_r)+\beta\left[\frac{\rho y_0}{4\theta}\left(1+k_1R\right)^2\frac{k_2}{k_1}\right]\left(\frac{\theta}{b}+\frac{(1-\theta)}{a}\right),$$

while the group of *low-talented* children receiving only basic education, i.e. children who cannot reach the required admission threshold even with the maximal top-up, consists of children with a talent lower than

$$\left.\widehat{\omega}_r^h\right|_{\alpha=\overline{\alpha}} = (1-\theta)(1-\lambda_r)-(1-\theta)\beta\left[\frac{\rho y_0}{4\theta}\left(1+k_1R\right)^2\frac{k_2}{k_1}\right]\left(\frac{a-b}{ab}\right).$$

The group of *medium-high-talented* children includes all children whose talent is $\omega \in \left[(1-\theta)(1-\lambda_r), \left.\widetilde{\omega}_r^h\right|_{\alpha=\overline{\alpha}}\right)$. These children achieve the required admission score s because the nurture effect and the investment in top-up. When no parents invest in private education, these children enter higher education without top-up. However, given that the parents' choice to invest in top-up depends on the quality of higher education, for each child within this group there exists a range of allocation rules, such that the access to higher education requires an investment in top-up. This interval is represented by the white are in Figure 4, whose size changes according to the level of child talent. When the quality of higher education corresponds to a level of α belonging to the white area, the competition for the higher education becomes stronger, as parents tend to increase their investment in top-up. Then, parents belonging to this group have to invest in private education to keep their child among the best θ children. On the other hand, for all levels of α outside the white region, the benefit of having a high-educated child declines, therefore parents with less talented children reduce their investment in private education and medium-high talented children enter higher education without top-up (compare black and white region in Figure 4 when $\omega \in [(1-\theta)(1-\lambda_r), \left.\widetilde{\omega}_r\right|_{\alpha=\overline{\alpha}})$). Thus, for parents belonging to this group the investment in top-up is a means of defense in the race for the higher education.

The last group consists of *medium-low-talented* children with a talent level $\omega \in \left[\left.\widehat{\omega}_r^h\right|_{\alpha=\overline{\alpha}}, (1-\theta)(1-\lambda_r)\right)$. These children need supplemental education in order to reach the required admission score s, thereby accessing to higher education. As for the group of medium-high talented children, for each child within this group there exists a range of allocation rule such that her parent has the incentive to invest in private education. This range corresponds to the levels of α within the white region. For these allocation rules, indeed, the income advantage of having a high-educated child is larger than the cost of the private education. That is, in the race for higher education, if these parents do not invest in private education, their children are excluded from higher education. Therefore, the investment in top-up is the means to compensate the gap between ability and the admission threshold.

2.6.2 Medium income inequality

The case of *medium initial inequality* refers to a scenario where all parents (rich and poor) have a disposable income before the maximal investment in top-up greater than the threshold $\overline{x}$, i.e. $x_r > x_p > \overline{x}$. However, after the choice to afford the maximal investment in supplemental education, the rich parents still enjoy of a disposable income larger than $\overline{x}$, while the disposable income of the poor parents falls below $\overline{x}$, i.e. $x_r - \widehat{e}_r > \overline{x} > x_p - \widehat{e}_p$, with the two levels of maximal top-up described by (13) and (15) for rich and poor parents respectively.

Compared with the scenario of high inequality, here, the poor parents have a larger maximal willingness to invest in private education, therefore given the limited number of positions available for higher education, the policy-maker has to increase the admission score s. By replacing the two maximal investments in top-up given by (13) and (15) into the equilibrium condition, one obtains that the equilibrium admission threshold, ensuring that only θ children are admitted to higher education with a medium level of income inequality, is

$$s^m = (1-\theta) + \beta Z \left(\frac{\theta}{b} + \frac{(1-\theta)}{a} \right) + \theta \lambda_r + (1-\theta)\, \beta \frac{(a-b)}{a} \left(x_p - \overline{x} \right), \tag{31}$$

where the superscript m stands for "medium inequality". As anticipated this equilibrium score is larger than the one chosen with a higher level of income inequality, that is $s^m - s^h = (1-\theta)\, \beta \frac{(a-b)}{a} \left(x_p - \overline{x} \right) > 0$. The minimum level of talent required to be "eligible" for higher education with the maximal top-up is

$$\widehat{\omega}_r^m = (1-\theta)(1-\lambda_r) - \beta(1-\theta)\left(\frac{a-b}{a}\right)\left(\frac{Z}{b} - (x_p - \overline{x})\right), \tag{32}$$

for a child from a rich family, and

$$\widehat{\omega}_p^m = (1-\theta) + \beta\theta\left(\frac{a-b}{a}\right)\left(\frac{Z}{b} - (x_p - \overline{x})\right) + \theta\lambda_r, \tag{33}$$

for a child with a poor parent, where $\widehat{\omega}_p^m > \widehat{\omega}_r^m$ given that $\frac{Z}{b} - (x_p - \overline{x}) > 0$ and $\lambda_r > 0$. That is, since rich parents have a larger willingness to invest in private education than poor parents, they can compensate with supplemental education a lower level of child talent. Therefore, the minimum level of talent required to enter higher education with top-up is lower for a child from a rich family than for a child with a poor parent. Lastly, the talent level of the marginal rich child admitted to higher education with the lowest top-up is

$$\widetilde{\omega}_r^m = (1-\theta)(1-\lambda_r) + \beta Z\left(\frac{\theta}{b} + \frac{(1-\theta)}{a}\right) + (1-\theta)\beta\frac{(a-b)}{a}(x_p - \overline{x}). \tag{34}$$

The graphical representation of $\widehat{\omega}_r^m$ and $\widetilde{\omega}_r^m$, to identify the four groups of voters, is similar to the illustration in Figure 4, with the only difference that all curves are shifted upwards by $(1-\theta)\beta\frac{(a-b)}{a}(x_p - \overline{x})$.

2.6.3 Low income inequality

The case with *low initial inequality* is such that all parents, rich and poor, have a disposable income, net of the maximal investment in private education, greater than the threshold $\overline{x}$. Thus, all parents have the same marginal utility of income and, hence, the same maximal willingness to invest in supplemental private education, which is equal to $\frac{Z}{b}$ (*Case A* in (13)).

Given that parents are homogenous in terms of the maximal top-up, the level of inequality of opportunity is lower compared to the case of high and medium inequality. Rich parents have only the nurture effect as channel to bequeath their status to their children. The larger the investments in supplemental education of poor parents, the stronger is the competition for the access to higher education. At the same time, the policy-maker has to increase the admission threshold s, to ensure that the share of students accessing higher education is still equal to θ. By replacing $\widehat{e}_p = \widehat{e}_r = \frac{Z}{b}$ into (23), one obtains that the equilibrium admission score with low income inequality is

$$s^{\ell} = (1-\theta) + \beta\frac{Z}{b} + \theta\lambda_r, \tag{35}$$

where the superscript ℓ stands for "low inequality". Given that children from poor families achieve better performances in the admission test compared with the case of high and medium inequality, it follows that $s^{\ell} > s^{m} > s^{h}$, otherwise the fraction of students to be admitted to higher education is larger than θ.

From (24) and (25) we observe that the minimum level of talent, such that the investment in top-up is profitable is defined as

$$\widehat{\omega}_r^{\ell} = (1-\theta)(1-\lambda_r), \tag{36}$$

for a child from a rich family, and as

$$\widehat{\omega}_p^{\ell} = (1-\theta)(1-\lambda_r) + \lambda_r, \tag{37}$$

for a child with a poor parent. Parents whose child talent is $\omega < \widehat{\omega}_i^{\ell}$ do not resort to private education, because this investment is wasteful since their child cannot achieve the required admission score s^{ℓ} even with the maximal top-up.

The threshold $\widetilde{\omega}_r$ denoting the talent level of the marginal child accessing higher education with the lowest top-up is

$$\widetilde{\omega}_r^{\ell} = (1-\theta)(1-\lambda_r) + \beta\frac{Z}{b}, \tag{38}$$

while for a children from poor a family the threshold $\widetilde{\omega}_p$ corresponds to the admission test score s^{ℓ}.

The two thresholds $\widehat{\omega}^{\ell}$ and $\widetilde{\omega}_r^{\ell}$ partitioning the voters into different subgroups are illustrated by Figure 5. Here, there are three groups of voter: parents with a *high-talented* child, parents with a *low-talented* child and parents with a *medium-talented* child. More specifically, the group with high-talented children includes all parents whose child's talent is $\omega > \widetilde{\omega}_r^{\ell}\Big|_{\alpha=\overline{\alpha}} = (1-\theta)(1-\lambda_r) + \frac{\beta}{b}\frac{\rho y_0}{4\theta}(1+k_1R)^2\frac{k_2}{k_1}$. These children are always admitted to higher education, regardless the allocation rule α. The group of low-talented children consists of parents with a child who is always excluded from higher education. It is interesting to note that, the ability level of the marginal child belonging to this group is constant with respect to α, therefore the size of this group is constant for all possible allocation rules. Finally, the group of medium-talented children includes all parents who need to invest in supplemental education in order to send their child to higher education. Children within this group have a level of talent $\widehat{\omega}_r^\ell < \omega \leq \widetilde{\omega}_r^\ell\Big|_{\alpha=\overline{\alpha}}$.

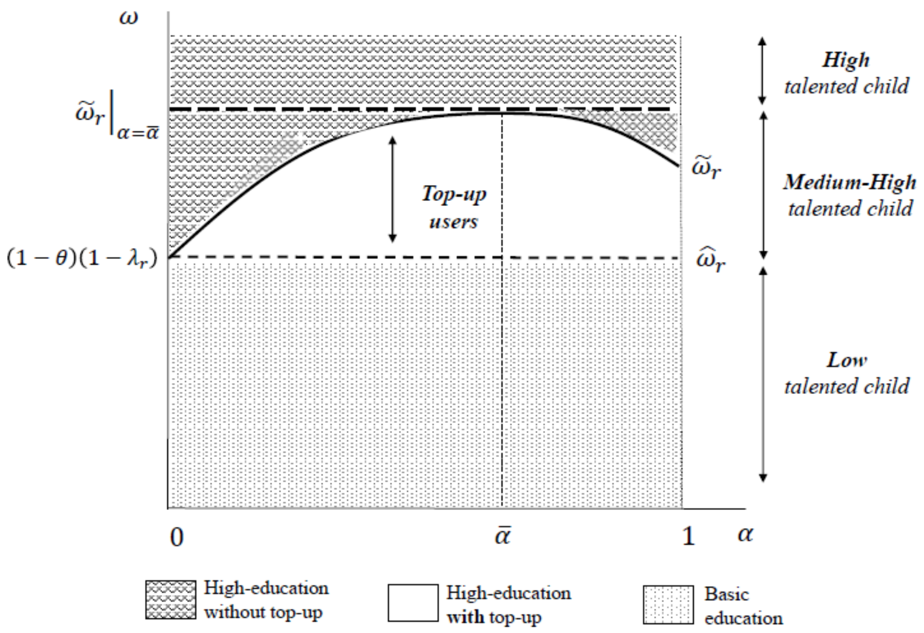

Fig. 5: Access to higher education with low inequality.

2.7 Preferences for the education policy

This section analyzes the preference for the allocation rule α of each group of rich parents. To simplify the exposition, here, we present the educational policy preferred by each group of voters, while all computations are relegated to Appendix A.

2.7.1 Parents with a high-talented child

A parent with a high-talented child (i.e. a child with a level of talent $\omega > \widetilde{\omega}_r|_{\alpha=\overline{\alpha}}$), who is always admitted to higher education without top-up, chooses the allocation rule α maximizing the following utility

$$u\left(x, y_r^{t+1}\right) = a\overline{x} + b\left(x - \overline{x}\right) + \rho y_0 \left(1 + k_1 g_B\right)\left(1 + k_2 g_U\right). \tag{39}$$

The next remark presents the optimal education policy supported by this group of parents.

Remark 2 *For a given education budget R and a level of access to higher education θ, the utility of a parent, whose child receives higher education without top-up, is maximized by*

$$\alpha_H = \frac{1}{2}\left(1 + \frac{1}{R}\left(\frac{1}{k_1} - \frac{\theta}{k_2}\right)\right), \tag{40}$$

if $\alpha_H \leq \min\{\overline{\alpha}, 1\}$ *or* $\alpha_H = \min\{\overline{\alpha}, 1\}$.

The optimal allocation rule for parents with a high-talented child is decreasing both in the size of the education budget R and in the level of access to higher education θ.

2.7.2 Parents with a medium-high talented child

This group of voters includes all parents, whose child talent is $\omega \in \left[(1-\theta)(1-\lambda_r), \widetilde{\omega}_r|_{\alpha=\overline{\alpha}}\right)$, who invest in top-up when the access to higher education is highly competitive. When α is low, these parents do not need to invest in private education, since their child achieves the required admission score with her ability. However, when α increases, the income advantage of having a high-educated child increases and all parents tend to invest more resources in private education. This increased competition makes the admission threshold s more demanding, therefore this group of parents need to resort to private education to safe their children access to higher education. Finally, given the complementarity between the two education levels, when α is extremely large, the income advantage associated with higher education decreases and the access become less competitive, so that this group of parents does not need to invest in private education (see Figure 4 and Figure 5). Let $\widetilde{\alpha}_1$ and $\widetilde{\alpha}_2$, with $\widetilde{\alpha}_1 < \widetilde{\alpha}_2$, denote the two marginal levels of α delimiting the set of the allocation rules such that the access to higher education is competitive and these parents need to invest in private education.

Then, the preference of this group of voters can be written as

$$u\left(x, y_r^{t+1}\right) = \begin{cases} a\overline{x} + b\left(x - \overline{x}\right) + \rho\left(y_0\left(1 + k_1 g_B\right)\left(1 + k_2 g_U\right)\right) & \text{if } \alpha \leq \widetilde{\alpha}_1 \text{or } \alpha \geq \widetilde{\alpha}_2 \\ a\overline{x} + b\left(x - e_j - \overline{x}\right) + \rho\left(y_0\left(1 + k_1 g_B\right)\left(1 + k_2 g_U\right)\right) & \text{if } \widetilde{\alpha}_1 < \alpha < \widetilde{\alpha}_2 \end{cases}, \tag{41}$$

where the term e_j in the second row of (41) represents the investment in top-up of the parent of the generic child with a level of talent ω_j. Since the investment in private education is aimed at achieving the required admission threshold, it follows that different levels of initial income inequality, by changing the admission threshold as described by Sections 2.6.1-2.6.3, impact on the optimal allocation rule of a parent investing in top-up.[12] The next remark presents the allocation rule maximizing the utility of a parent investing in top-up, i.e. the utility level in the second row of (41).

Remark 3 *When the level of initial income inequality is either high or medium, the allocation rule that maximizes the utility of a parents resorting to private education, for a given education budget R and a level of access to higher education θ, is either:*

$$a_T = \frac{1}{2}\left(1 + \frac{1}{R}\left(\frac{1}{k_1} - \frac{\theta}{k_2\phi}\right)\right), \tag{42}$$

with $0 \leq \alpha_T \leq \min\left\{\overline{\alpha}, 1\right\}$ *or* $\alpha_T = \min\left\{\overline{\alpha}, 1\right\}$ *if* $\frac{1}{k_1} > \frac{\theta}{k_2\phi}$; *or*

$$a_T = 0$$

if $R < \frac{\theta}{k_2\phi} - \frac{1}{k_1}$.

When the level of initial income inequality is low the optimal allocation rule is

$$a_T = 0.$$

The child level ω of innate talent does not affect the allocation rule a_T. By comparing (40) and (42), one may note that parents with a high-talented child, admitted to higher education without top-up, prefer educational policies more generous towards higher education, than parents who invest in private education, i.e. $a_H > a_T$,

[12] Recall that in this analysis we assume that rich parents always have a disposable income, net of the maximal investment in private education, greater than the threshold $\overline{x}$. Then, to consider different levels of income inequality, we vary the level of the disposable income of the poor, as described by Sections 2.6.1, 2.6.2 and 2.6.3.

given that $\phi < 1$.

Figure 6 illustrates the utility of parents with a medium high-talented child when the level of initial income inequality is either high or medium (panel 6a) and low (panel 6b). In both panel, each curve is labeled with the level of child talent, with $\omega > \omega_3 > \omega_2 > \omega_1$.

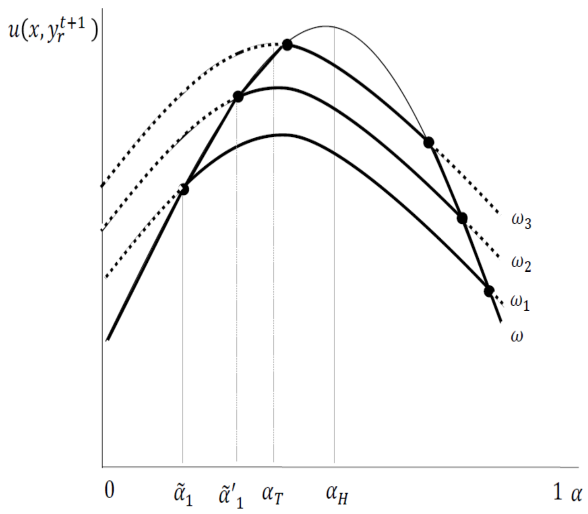

Fig. 6a: Preference with high ineq.

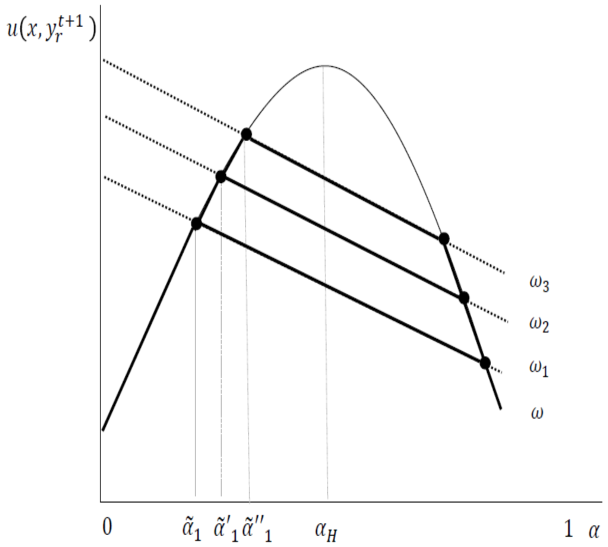

Fig. 6b: Preferences with low ineq.

The utility of each parent is obtained by combining the two rows of (41). More specifically, the first row corresponds to the curve ω, which is the utility obtained when the child receives higher education without top-up. As shown by both panels, this curve is single peaked with the peak $\alpha = \alpha_H$. The second row of (41), instead, is illustrated by the dotted curves, which can be: either *(i))* single peaked at $\alpha = \alpha_T$, if the level of income inequality is high or medium (panel 6a); or *(ii))* decreasing in the allocation rule, if the level of income inequality is low (panel 6b).

When the level of the child talent increases, the dotted curve shift upwards, because the amount of top-up necessary to achieve the admission threshold s decreases. The three bold curves represent the utility of three parents with a medium-high talented child. All these three curves are single peaked, however, the peak changes according to the level of $\widetilde{\alpha}_1$, i.e. the marginal level of the investment in higher education, such that parents of this group need to resort to private education. Lemma 2 describes the preference of a parent with a medium-high talented child, when the level of initial income inequality is either high or medium and the differential in the maximal top-up between rich and poor parents is measured by the difference between (13) and (17) or between (13) and (15) respectively.

Lemma 2 *When the level of initial income inequality is such that rich parents have a lower marginal utility of net income and are willing to afford a higher maximal investment in private education than poor parents, the optimal allocation rule of a parent with a medium-high talented child is:*

i. $a^* = \alpha_T$ *if* $\widetilde{\alpha}_1 \leq \alpha_T$;

ii. $a^* = \widetilde{\alpha}_1$ *if* $\alpha_T < \widetilde{\alpha}_1 < \alpha_H$;

iii. $a^* = \alpha_H$ *if* $\widetilde{\alpha}_1 \geq \alpha_H$.

Next lemma, instead, summarizes the preference of a generic parent with a medium-high talented child when the level of initial income inequality is low and there is no difference in the maximal investment in private education chosen by rich and poor parents respectively.

Lemma 3 *When the level of initial income inequality is so low that all parents, rich and poor, experience the same marginal utility of net income, the optimal education policy for a generic parent with a medium-high talented child is:*

i. $a^* = \widetilde{\alpha}_1$ *if* $0 \leq \widetilde{\alpha}_1 < \alpha_H$;

ii. $a^* = \alpha_H$ *if* $\widetilde{\alpha}_1 \geq \alpha_H$.

2.7.3 Parents with a medium-low talented child

The preference of the generic parent with a medium-low talented child, i.e. a child with a level of talent $\omega \in \left[\widehat{\omega}_r|_{\alpha=\overline{\alpha}}, (1-\theta)(1-\lambda_r)\right)$, can be formalized as follows

$$u\left(x, y_i^{t+1}\right) = \begin{cases} a\overline{x} + b\left(x - \overline{x}\right) + \rho\left(y_0\left(1 + k_1 g_B\right)\right) & \text{if } \alpha \leq \widetilde{\alpha}_1 \text{or } \alpha \geq \widetilde{\alpha}_2 \\ a\overline{x} + b\left(x - e_j - \overline{x}\right) + \rho\left(y_0\left(1 + k_1 g_B\right)\left(1 + k_2 g_U\right)\right) & \text{if } \widetilde{\alpha}_1 < \alpha < \widetilde{\alpha}_2 \end{cases} . \tag{43}$$

When the investment in higher education is extremely low (i.e. $\alpha \leq \widetilde{\alpha}_1$) or is very high (i.e. $\alpha \geq \widetilde{\alpha}_2$), parents belonging to this group prefer not to invest in private education, so that their child receives only basic education. The next remark presents the optimal allocation rule associated with the first row of (43).

Remark 4 *For a given education budget R and a level of access to high education θ, the optimal allocation rule maximizing the utility of a parent, whose child receives only basic education, is*

$$\alpha^* = 0. \tag{44}$$

On the other hand, when $\alpha \in (\widetilde{\alpha}_1, \widetilde{\alpha}_2)$ these parents decide to invest in private education, because the cost of this investment is more than compensated by the benefit of having a high-educated child. The optimal allocation rule maximizing the utility level in the second row of (43) is described by Remark 3.

Given the result in Remark 3, it follows that when the level of income inequality is so low and rich and poor parents have the same marginal utility of income, all parents with a medium-low talented child prefer to allocate the entire education budget to basic education, that represents a sort of insurance for the child's future income. Figure 7 illustrates the preferences of this group of parents when the level of initial income inequality is either high or medium.

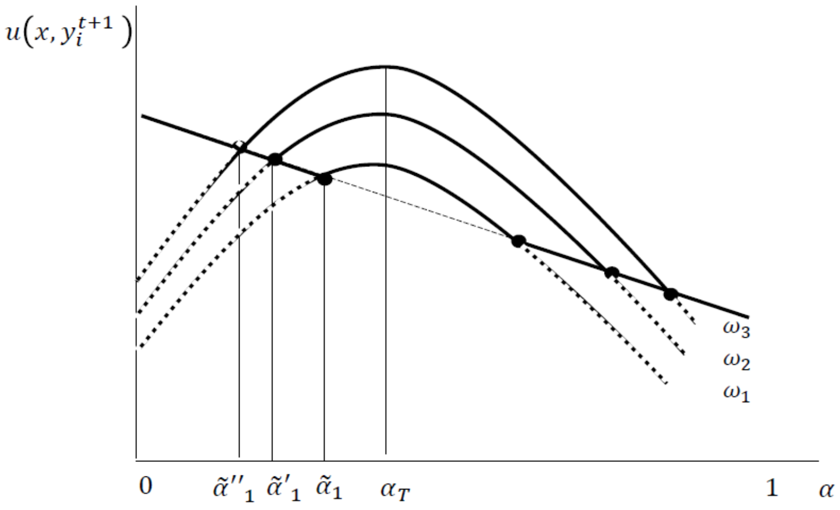

Fig. 7: Utility of parents with a medium-low talented child.

As in Figure 6, each curve is labeled with the level of child's talent, with $\omega_3 > \omega_2 > \omega_1$. The decreasing line represents the utility when the child receives only basic education (i.e. first row of 43), while the inverted-U shaped curve is the utility of three generic parents investing in private education in order to have a high educated child (i.e. second row of 43). The utility of each of these parents corresponds to a specific bold curve, which is obtained by combining the two rows of (43). More specifically, when the investment in higher education is given by $\alpha \notin (\widetilde{\alpha}_1, \widetilde{\alpha}_2)$, the utility corresponds to the decreasing line, while for all the allocation rules $\widetilde{\alpha}_1 < \alpha < \widetilde{\alpha}_2$ the utility is given by the inverted-U shaped curve. If the allocation rule is either $\alpha = \widetilde{\alpha}_1$ or $\alpha = \widetilde{\alpha}_2$,the parent is indifferent between having a low-educated or a high-educated child, by investing the maximal top-up.

When the level of a child innate talent decreases, the inverted-U shaped curve shifts downward, then the utility associated with $\alpha = \alpha_T$ may be lower than the utility obtained when the entire education budget is allocated to basic education (see Figure 7, the utility of parent whose child talent is ω_1). For each allocation rule $\alpha \in (0, 1)$ there exists a level of talent $\overline{\omega}_r$ denoting the marginal child for which the cost of top-up is equal to the benefit associated with higher education. This level, whose derivation is illustrated by Appendix B, is defined as

$$\overline{\omega}_r^h = (1-\theta)(1-\lambda_r) + \frac{\beta}{b}(\rho y_0 \alpha R k_1 - Z\phi), \tag{45}$$

when the initial income inequality is high, and as

$$\overline{\omega}_r^m = (1-\theta)(1-\lambda_r) + (1-\theta)\beta\left(\frac{a-b}{a}\right)(x_i - \overline{x}) + \frac{\beta}{b}(\rho y_0 \alpha R k_1 - Z\phi), \tag{46}$$

when the level of initial income inequality is medium, while with a low level of initial income inequality becomes as

$$\overline{\omega}_r^\ell = (1-\theta)(1-\lambda_r) + \frac{\beta}{b}\rho y_0 \alpha R k_1. \tag{47}$$

The next Lemma summarizes the preference for the education policy of a parent with a medium-low talented child.

Lemma 4 *When the level of initial income inequality is such that rich and poor parents are willing to afford a different maximal investment in supplemental education,* *the optimal allocation rule chosen by the generic parent with a medium-low talented child is:*

i. $a^* = \alpha_T$ *if* $\overline{\omega}^i < \omega < (1-\theta)(1-\lambda_r)$;

ii. $a^* = 0$ *if* $\omega < \overline{\omega}^i_r$.

where $i = h, m$ *denotes the corresponding level of initial income inequality. When the level of initial income inequality, instead, is low and there is no difference between the maximal top-up of rich and poor parents, the optimal allocation rule for the parents with a medium-low talented child is* $a^* = 0$ *.*

2.7.4 Parents with a low-talented child

A parent with a low-talented child (i.e. a child with a level of talent $\omega < \widehat{\omega}_r|_{\alpha=\overline{\alpha}}$) who cannot reach the required admission threshold even with the maximal top-up, decides to allocate the entire education budget to basic education. The utility of this group of parents, indeed, corresponds to the first row of (43), which is maximized when $\alpha = 0$, as reported in Remark 4.

3 The political-economy equilibrium

Section 2.7 has introduced the allocation rules candidate to be the political-economy equilibrium under majority voting. The potential candidates are: *i)* $\alpha = 0$, which is supported by the parents whose child receives only basic education[13]; *ii)* $\alpha = \alpha_T$, that is the optimal choice of the parents with a medium-talented child, who resort to top-up when the investment in higher education is sufficiently large; *iii)* $\alpha \in (\alpha_T, \alpha_H)$, which represents the set of the alternatives voted by parents with a medium-high talented child, who needs the investment in top-up when the competition is high; *iv)* $\alpha = \alpha_H$, namely the optimal education policy preferred by the parents with a high-talented child admitted to higher education without top-up.

[13] This group of voters includes: *i)* all parents with a low-talented child (i.e. a child with a level of talent $\omega \leq \widehat{\omega}_r$) who cannot reach the required admission score even with the maximal top-up; *ii)* a fraction of the group of parents with a medium-low talented child (i.e. a child with a level of talent $\omega \leq \overline{\omega}$), for which the cost of top-up to reach the admission score is larger than the income benefit of higher education.

The optimal allocation rule for the society is the α chosen under majority voting, which corresponds to the choice of the median voter. By using the optimal allocation rules given by (40) and (42), we define three talent threshold levels, that are useful to derive the political economy equilibrium. The first threshold, denoted by $\left.\overline{\omega}_r^i\right|_{\alpha=\alpha_T}$, is obtained by replacing (42) into the definition of $\overline{\omega}_r$, i.e. the ability level of the marginal child for which the investment in top-up is profitable. This threshold represents the share of parents who prefer $\alpha = 0$ to any other alternative $\alpha \geq \alpha_T$.

The second threshold, labeled $\left.\widetilde{\omega}_r^i\right|_{\alpha=\alpha_T}$, corresponds to the talent level of the marginal child accessing higher education with the lowest level of top-up, when the investment in higher education is α_T. This threshold identifies the share of parents supporting an allocation rule $\alpha \in \{0, \alpha_{TU}\}$. Lastly, the third threshold, denoted by $\left.\widetilde{\omega}_r^i\right|_{\alpha=\alpha_H}$, is the talent level of the marginal child admitted to higher education with the lowest top-up, when the allocation rule is α_H (which is defined by (40)). This thresholds represents the share of parents voting for $\alpha < \alpha_H$.

The superscript i of each of these three thresholds denotes the level of the initial income inequality, with $i = h, m, \ell$. When the level of initial inequality between rich and poor parents changes, the curves associated with the thresholds $\widetilde{\omega}_r^i$ and $\overline{\omega}_r^i$ are shifted, then the share of parents supporting the alternative allocation rule changes. Figure 8 and Figure 9 illustrate these shares when the level of the initial income inequality is respectively high and low[14]. Given that the highest allocation rule candidate to be the political economy equilibrium is α_H, to simplify the representation, we consider in both figures all the allocation rules $\alpha \in [0, \overline{\alpha}]$, where $\overline{\alpha} = \frac{1}{2}\left(1 + \frac{1}{k_1 R}\right) > \alpha_H$. This value, derived in Section 2.5, represents the level of the investment in higher education that maximizes the income advantage of having a high-educated child, i.e. term Z entering in the definition of the thresholds $\widetilde{\omega}_r^i$, $\widehat{\omega}_r^i$ and $\overline{\omega}_r^i$.

[14] The graphical representation of the case with a medium level of initial income inequality is similar to the illustration in Figure 8. The only difference is that all curves are shifted upwards by $(1-\theta)\,\beta\frac{(a-b)}{a}\,(x_p - \overline{x})$.

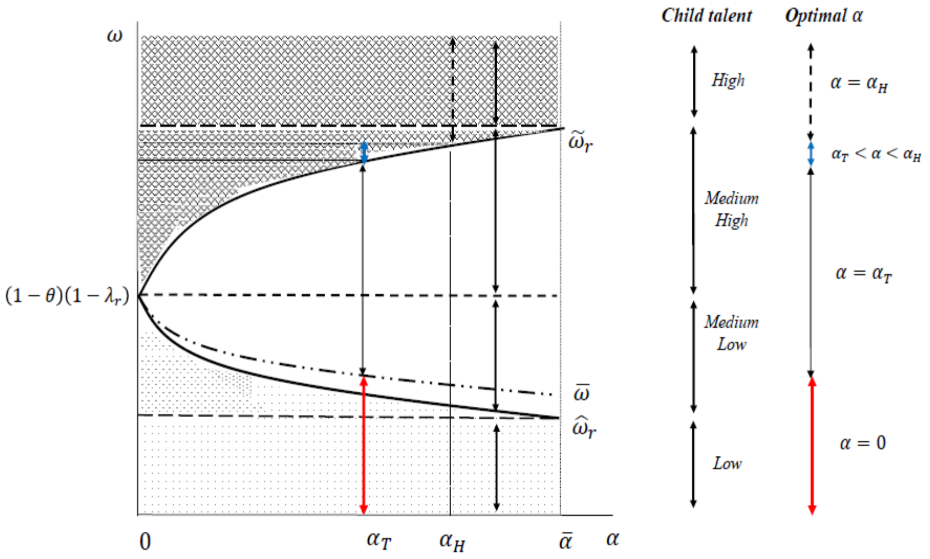

Fig. 8: Preferences for α with high inequality.

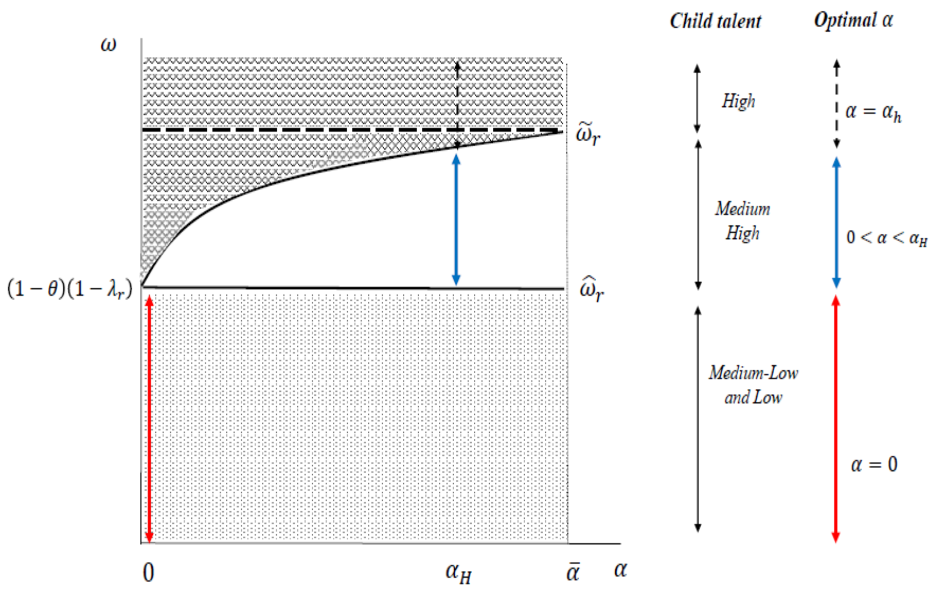

Fig. 9: Preferences for α with low inequality.

Let α_{MV} denote the majority voting allocation rule, then the political economy equilibrium is described by the next proposition.

Proposition 2 *When the level of initial income inequality is such that rich parents are willing to afford a larger investment in top-up than poor parents, the allocation rule chosen under majority voting is:*

i. $\alpha_{MV} = 0$ *if* $\left.\overline{\omega}_r^i\right|_{\alpha=\alpha_T} > \frac{1}{2}$;

ii. $\alpha_{MV} = \alpha_T$ *if* $\left.\overline{\omega}_r^i\right|_{\alpha=\alpha_T} \leq \frac{1}{2} < \left.\widetilde{\omega}_r^i\right|_{\alpha=\alpha_T}$;

iii. $\alpha_{MV} \in (\alpha_T, \alpha_H)$ *if* $\left.\widetilde{\omega}_r^i\right|_{\alpha=\alpha_T} \leq \frac{1}{2} \leq \left.\widetilde{\omega}_r^i\right|_{\alpha=\alpha_H}$;

iv. $\alpha_{MV} = \alpha_H$ *if* $\left.\widetilde{\omega}_r^i\right|_{\alpha=\alpha_H} < \frac{1}{2}$,

with $i = h, m$ *if the level of initial income inequality is respectively high or medium.*

When the level of initial income inequality is low and all parents, rich and poor, have the same maximal investment in top-up, the allocation rule α_T is equal to zero (see Appendix A). Then, the political economy equilibrium can be presented as in the following proposition.

Proposition 3 *With a low level of income inequality, such that the differential in the maximal top-up between rich and poor parents is zero, the political economy equilibrium is:*

i. $\alpha_{MV} = 0$ *if* $(1-\theta)(1-\lambda_r) > \frac{1}{2}$;

ii. $\alpha_{MV} \in (0, \alpha_H)$ *if* $(1-\theta)(1-\lambda_r) \leq \frac{1}{2} \leq \left.\widetilde{\omega}_r^\ell\right|_{\alpha=\alpha_H}$;

iii. $\alpha_{MV} = \alpha_H$ *if* $\left.\widetilde{\omega}_r^\ell\right|_{\alpha=\alpha_H} < \frac{1}{2}$.

3.1 Comments and implications

The results presented by Proposition 2 and Proposition 3 show that the share α allocated to higher education is related with the level of initial income inequality and with the level of social exchange mobility.

When initial income inequality is either high or medium and rich parents, having a lower marginal utility of income, invest more resources in private education, the conditions to have $\alpha > 0$ as the political economy equilibrium are less demanding compared to the case with low initial income inequality.

Moreover, the higher is the level of initial income inequality, the higher is the level of inequality of opportunity in the access to higher education. When rich parents invest more resources in private education, their children have a larger probability to be admitted to higher education than children from a poor family. Given that, the larger is the share of children from a rich family accessing to higher education, the larger is the support for an allocation rule $\alpha > 0$, it follows that the investments in an "elitist" higher education are more likely when the level of social exchange mobility is low.

Finally, it is interesting to note that all the allocation rules $\alpha > 0$ are decreasing in the size of the education budget R (see (40) and (42)). Given that $R = \tau\mu$, it follows that for a given tax rate, the larger the average income of the economy, the lower is the share allocated to higher education. By considering the average income as a measure of the country's level of development, we have that the larger the level of development, the lower is the relative per-student expenditure in higher education, measured by ratio $\frac{g_U}{g_B}$.

4 Social Mobility and Dynamics

Based on previous results we discuss here the link between the educational policy and the level of social mobility, and highlight some dynamic effects of this model.

4.1 Social mobility

Recall that only rich parents are called to vote for the educational policy. Therefore, as shown by Proposition 2 and Proposition 3, respectively for the case with high (or medium) and low initial income inequality, the majority voting educational policy depends on the level of social exchange "immobility at the top", which is measured by the share of rich parents whose child is admitted to higher education. More specifically, for a positive allocation rule (i.e. $\alpha > 0$) the share of children from a rich family entering higher education and therefore becoming rich is defined as

$$Q_U^i = 1 - F\left(\widehat{\omega}_r^i\right). \tag{48}$$

This share can be interpreted as a measure of social immobility at the top. On the other hand, the share of children with a rich parent who receive only basic education is

$$Q_B^i = F\left(\widehat{\omega}_r^i\right), \tag{49}$$

which represents the level of downward social mobility. The two shares (48) and (49) are defined by using the threshold $\widehat{\omega}_r^i$ defined in Section 2.6 for different levels of initial income inequality. This threshold represents the talent level of the marginal child, from a rich family, entering higher education with the maximal top-up. Given the definitions of the threshold $\widehat{\omega}_r^i$ associated with the different levels of initial income inequality in (28), (32) and (36), it follows that $\widehat{\omega}_r^\ell > \widehat{\omega}_r^m > \widehat{\omega}_r^h$. The higher the initial income inequality, the lower is the level of social exchange mobility, as the share of children from a rich background admitted to higher education increases i.e., $Q_U^\ell < Q_U^m < Q_U^h$.

Similarly, by using the talent threshold $\widehat{\omega}_p^i$, the share of children with a low-educated parent who are admitted to higher education is defined as

$$q_U^i = 1 - F\left(\widehat{\omega}_p^i\right). \tag{50}$$

While the share of children from a poor background receiving only basic education is

$$q_B^i = F\left(\widehat{\omega}_p^i\right). \tag{51}$$

The two shares in (50) and (51) represent respectively a measure of the upward exchange social mobility and of the immobility at the bottom. Given the definitions of the threshold $\widehat{\omega}_p^i$ for the different levels of initial income inequality in (29), (33) and (37), it follows that $\widehat{\omega}_p^\ell < \widehat{\omega}_p^m < \widehat{\omega}_p^h$. Thus, the higher the level of initial income inequality, the lower is the level of upward social mobility for children from a poor family, i.e. $q_U^\ell > q_U^m > q_U^h$. Moreover, for each level of initial income inequality $\widehat{\omega}_p^i > \widehat{\omega}_r^i$, by using (24) and (25) the difference between these two thresholds can be written as

$$\widehat{\omega}_p^i - \widehat{\omega}_r^i := \lambda_r + \beta\left(\widehat{e}_r - \widehat{e}_p\right) > 0. \tag{52}$$

That is, the inequality of opportunity in the access to higher education between children from rich and poor families has two different sources: the nurture effect (λ_r) and the different possibilities of resorting to supplemental private education $(\widehat{e}_r - \widehat{e}_p)$, which depend on the level of initial income inequality. The exchange social mobility is summarized by Table 2, where each panel is associated with a specific level of initial income inequality. It is interesting to note that, in each panel the presence of the inequality of opportunity implies that the share of children from a rich [poor] family inheriting the social status of their parent is larger than θ $[1-\theta]$.

Table 2: Social Exchange Mobility and Initial Income Inequality

Panel A: *High initial income inequality*

Social Class	y_p^{t+1}	y_r^{t+1}	n
y_p^t	$(1-\theta)\times\left((1-\theta)+\theta\left(\left(\lambda_r+\pi^h\right)_1\right)\right)$	$(1-\theta)\times\left(\theta\left(1-\left(\lambda_r+\pi^h\right)_1\right)\right)$	$1-\theta$
y_r^t	$\theta\times(1-\theta)\left(1-\left(\lambda_r+\pi^h\right)_1\right)$	$\theta\times\left(\theta+(1-\theta)\left(\left(\lambda_r+\pi^h\right)_1\right)\right)$	θ
n	$1-\theta$	θ	1

where $\left(\lambda_r+\pi^h\right)_1 := \min\left\{1, \lambda_r+\pi^h\right\}$ and $\pi^h = \beta\frac{Z}{b}\left(\frac{a-b}{a}\right)$.

Panel B: *Medium initial income inequality*

Social Class	y_p^{t+1}	y_r^{t+1}	n
y_p^t	$(1-\theta)\times\left((1-\theta)+\theta\left(\left(\lambda_r+\pi^m\right)_1\right)\right)$	$(1-\theta)\times\left(\theta\left(1-\left(\lambda_r+\pi^m\right)_1\right)\right)$	$1-\theta$
y_r^t	$\theta\times(1-\theta)\left(1-\left(\left(\lambda_r+\pi^m\right)_1\right)\right)$	$\theta\times\left(\theta+(1-\theta)\left(\left(\lambda_r+\pi^m\right)_1\right)\right)$	θ
n	$1-\theta$	θ	1

where $\left(\lambda_r+\pi^m\right)_1 := \min\left\{1, \lambda_r+\pi^h\right\}$ and $\pi^m = \beta\left(\frac{a-b}{a}\right)\left(\frac{Z}{b}-(x_p-\overline{x})\right)$.

Panel C: *Low initial income inequality*

Social Class	y_p^{t+1}	y_r^{t+1}	n
y_p^t	$(1-\theta)\times\left((1-\theta)+\theta\lambda_r\right)$	$(1-\theta)\times\theta\left(1-\lambda_r\right)$	$1-\theta$
y_r^t	$\theta\times(1-\theta)\left(1-\lambda_r\right)$	$\theta\times\left(\theta+\lambda_r(1-\theta)\right)$	θ
n	$1-\theta$	θ	1

Recall that the term Z appearing in Panel A and Panel B of Table 2, is positively related with the allocation rules candidate to the political economy equilibrium under majority voting. Therefore, with either a high or medium level of initial income inequality, the threshold $\widehat{\omega}_r^i$ decreases when the share of the education budget allocated to higher education increases (see Figure 8). That is, the more generous the allocation rule towards higher education, the larger is the level of social immobility at the top. On the other hand, given the definition of $\widehat{\omega}_p^i$, the larger the share of the education budget allocated to higher education, the higher is the level of social immobility at the bottom. When rich and poor parents exhibiting different maximal top-up, i.e. when the level of initial income inequality is either high or medium, larger ratios between per-student expenditure in higher and basic education tend to be associated with a low level of exchange social mobility, as the share of children inheriting the parental social status increases.

When the level of initial income inequality is low and all parents have the same maximal top-up, the level of social exchange mobility does not depend on the allocation rule (see Panel C).

The level of social immobility (I) associated with a generic level i of initial income inequality is defined as $I^i = 2\theta(1-\theta)\min\{1, \lambda_r + \pi^i\}$, with $i = h, m, \ell$, and $\pi^\ell = 0$.

4.2 Dynamics

The dynamics of the educational policy is related to the evolution of the average income of the society that, for a given tax rate τ, affects the size of the education budget R. For a given allocation rule α, the average income of the children generation is defined as

$$\mu^{t+1} = y_0\left(1 + k_1\left(1-\alpha^t\right)\tau^t\mu^t\right)\left(1 + k_2\alpha^t\tau^t\mu^t\right). \tag{53}$$

For exposition purposes we set now $k_1 = k_2 = k$ and assume that the allocation rule decided by the society is $\alpha^t = \alpha_H = \frac{1}{2}\left(1 + \frac{1}{Rk}(1-\theta)\right)$. Given these assumptions, the average income in (53), after some manipulations becomes as

$$\mu^{t+1} = \frac{y_0}{4}\left(\left(2 + k\tau^t\mu^t\right)^2 - (1-\theta)^2\right). \tag{54}$$

Figure 10 illustrates the relationship in (54), which is linear for all levels of the initial average income $\mu^t \leq \frac{1-\theta}{k\tau}$, where the cutoff level $\frac{1-\theta}{k\tau}$ is given by $\alpha_H \leq 1$. When the initial average income is $\frac{1-\theta}{k\tau}$ the relationship becomes convex.

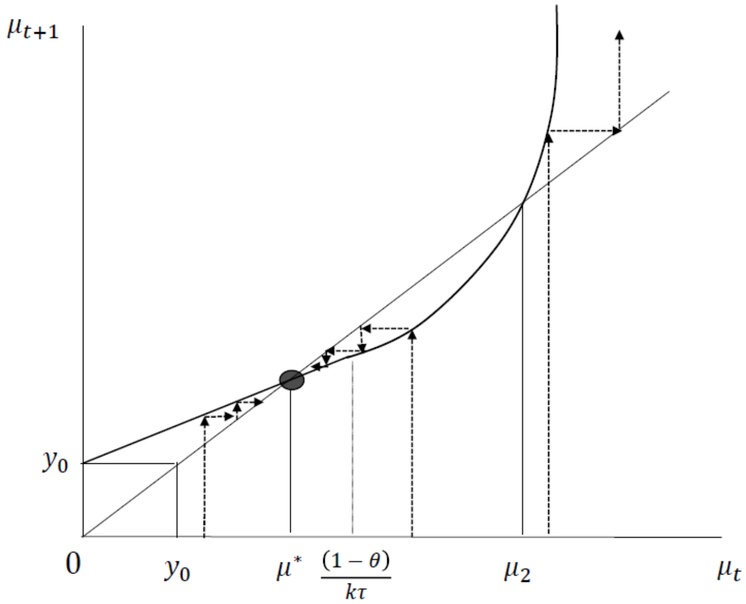

Fig. 10: Dynamics.

Figure 10 shows that there are two alternative patterns that an economy may follow. The first pattern characterizes an economy whose initial average income is larger than μ_2. Under this scenario, the average income of the society tends to grow over time. The second pattern involves the convergence towards μ^* for all economies starting below μ_2.

Given the dynamics of the average income illustrated by Figure 10, it is possible to analyze how the educational policy evolves over time. More specifically, when the level of the initial income inequality is high and the political economy equilibrium is α_H, if the initial average income is larger than μ_2, the economy grows over time, because the income of both rich and poor parents increase. Then, the growth of the income of poor parents implies that their disposable income (either before or post the maximal investment in top-up) may become larger than the threshold $\overline{x}$. In this case the poor, having a lower marginal utility of net income, increase their maximal investment in top-up. Therefore, the growth of the average income and the lower differential of in terms of maximal top-up between poor and rich parents, imply that the condition to continue to have α_H as the political economy equilibrium, may no longer be satisfied. In particular this condition presented at point *(iv)* of Proposition

2, can be rewritten as

$$(1-\theta)(1-\lambda_r) < \frac{1}{2} - \left(\frac{1}{\theta a} + \frac{a-b}{ab}\right)\frac{\beta\rho y_0}{4}\left((Rk+1)^2 - \theta^2\right), \tag{55}$$

where the growth of the average income increases the term R (we assume that marginal tax rate is constant), while the parameters a and b capture the effect of variations in the marginal utility of the poor. That is, when the average income grows and poor have more resources to invest in private education, the second term on the RHS of condition (55) becomes larger. Therefore, if the term λ_r (i.e. the nurture effect) is not extremely high, the political economy equilibrium will continue to be α_H if the level of elitism of the higher education declines, i.e. if θ increases.

It is interesting to note that, the growth of the average income reduces the level of all the allocation rules candidate to be the political economy equilibrium (see (40) and (42)). That is, the higher the average income, the lower is the level of α_H, (recall that $\alpha_H \to \frac{1}{2}$when $R \to \infty$). Therefore, one economy with an initial income larger than μ_2 experiences a continuous growth of the average income. This reduces the inequality between income classes and the elite has to reduce the elitism of the higher education in order to keep their preferred policy as the political economy equilibrium. In addition, the growth of the average income reduces the level of all the alternative allocation rules, then the ratio of per-student expenditure tend to decline over time. Thus, developed economies (whose initial average income is larger than μ_2) are characterized by low levels of both income inequality and relative expenditure in higher education.

When the initial income of the economy is below μ_2, there is a convergence towards μ^*. In particular, if the initial income is $\mu \in (y_0, \mu^*)$ (i.e. low developed economy) the average income grows until μ^*. For example, with a high level of initial income inequality and a share α_H of the education budget allocated to higher education, the dynamics implications of the growth of the average income are the same as those associated with the case of $\mu > \mu_2$. That is, the average income increases and the poor have more resources to invest in private education. Then, the condition to have α_H as the political economy equilibrium requires that the elite has to make the access to higher education less elitist. However, different than the case of a developed economy (i.e. $\mu > \mu_2$), here, the economy converge towards a steady state with a low average income and then a high ratio of per-student expenditures $\frac{g_U}{g_B}$. In addition, a large share of the education budget allocated to higher education is associated with a low level of exchange mobility, because a large share of children from a rich family inherits their parent's income class.

When the initial level of development is $\mu > \mu^*$ (medium developed economy), the average income tends to decrease over time. If the allocation rule is α_H, this decreasing pattern implies that the income inequality increases and the differential in the maximal top-up does not declines. Hence, the condition (55) continues to be satisfied and the society converges towards an equilibrium characterized by a low-quality basic education and a good-quality elitist higher education.

5 Concluding remarks

This paper proposes a political economy explanation of the cross-country differences observed in educational policies. We consider the case of less-developed economies, where only a rich elite has political power and has to decide how to allocate a given public budget across two education levels, i.e. basic and higher education. Our results reveal that the educational policy implemented under majority voting is related to the country's levels of initial development and income inequality.

When income inequality is high and development is low the rich elite implements a policy that allocates a large share of public budget to higher education. This result is consistent with the empirical evidence showing that less developed countries tend to have larger relative schooling expenditure, measured by the ratio between per capita investments in higher education over investments in basic education. In addition, this scenario is associated with a low level of exchange social mobility, as the majority of rich parents bequeath their social class to their children.

With regard to the dynamics, the analysis presented in this paper considers some reference cases, i.e. high inequality and initial allocation rule α_H, and discusses how the elite may implement its preferred policy over time and the implication of this policy. The main conjecture is that, when the economy grows the elite has to reduce the level of elitism of the higher education, otherwise the majority will choose a different educational policy. How the level of elitism is decided by the society and the analysis of its implications would be a topic for future research.

Appendix A: Preferences for the education policy

This Appendix provides all the computations to derive the optimal allocation policy for the different groups of voters presented in Section 2.7.

Parents with a high-talented child

Recall that the utility of a rich parent whose child enters higher education without top-up is

$$u\left(x, y_r^{t+1}\right) = a\overline{x} + b\left(x - \overline{x}\right) + \rho y_0 \left(1 + k_1 g_B\right)\left(1 + k_2 g_U\right). \tag{56}$$

The first order condition (hereafter FOC) with respect to α is

$$\frac{\partial u\left(x, y_r^{t+1}\right)}{\partial \alpha} = \rho y_0 \left(\frac{dg_B}{d\alpha} k_1 \left(1 + k_2 g_U\right) + \left(1 + k_1 g_B\right)\frac{dg_U}{d\alpha} k_2\right) = 0. \tag{57}$$

By replacing the definition of the per-student expenditure in basic and higher education, given by (1) and (2) respectively, the previous FOC (**??**) can be rewritten as

$$-Rk_1\left(1 + k_2 \frac{\alpha R}{\theta}\right) + \left(1 + k_1\left(1 - \alpha\right)R\right)k_2 \frac{R}{\theta} = 0,$$

Then, by solving for α, one obtains that the optimal allocation rule chosen by a parent with a high-talented child is

$$\alpha_H = \frac{1}{2}\left(1 + \frac{1}{R}\left(\frac{1}{k_1} - \frac{\theta}{k_2}\right)\right).$$

Note that $\alpha_H > 0$ given that $\frac{1}{k_1} > \frac{\theta}{k_2}$. While $\alpha_H < 1$ if $R > \left(\frac{1}{k_1} - \frac{\theta}{k_2}\right)$. However, $\alpha_H \leq \overline{\alpha} = \frac{1}{2}\left(1 + \frac{1}{Rk_1}\right)$, where $\overline{\alpha}$ is the allocation rule that maximizes the income advantage Z of having a high educated child.

Parents with a medium-talented child

The utility of a rich parent investing in supplemental private education is

$$u\left(x - e_j, y_r^{t+1}\right) = a\overline{x} + b\left(x - e_j - \overline{x}\right) + \rho y_0 \left(1 + k_1 g_B\right)\left(1 + k_2 g_U\right), \tag{58}$$

where the term e_j denotes the top-up needed for the child with a level of talent ω_j to achieve the required admission score threshold s^i. Since the admission threshold s^i changes with the level of initial income inequality, as described in Sections 2.6.1, 2.6.2 and 2.6.3, here, we consider the case with high, medium and low initial income inequality separately.

High initial income inequality. When the level of initial income inequality is high, the required admission score is

$$s^h = (1-\theta) + \beta Z \left(\frac{\theta}{b} + \frac{(1-\theta)}{a} \right) + \theta \lambda_r, \tag{59}$$

then a parents whose child's talent is ω_j, in order to have a high-educated child, has to purchase the following amount of top-up

$$e_j^h := \frac{1}{\beta} \left(s^h - \lambda_r - \omega_j \right) = \frac{1}{\beta} \left((1-\theta)(1-\lambda_r) + \beta Z \left(\frac{\theta}{b} + \frac{(1-\theta)}{a} \right) - \omega_j \right), \tag{60}$$

recall that the superscript h stands for high inequality. Given the definition of Z (equation (11)), the top-up investment can be rewritten as

$$e_j^h = \frac{1}{\beta} \left((1-\theta)(1-\lambda_r) + \beta \rho k_2 g_U y_0 (1 + k_1 g_B) \left(\frac{\theta}{b} + \frac{(1-\theta)}{a} \right) - \omega_j \right). \tag{61}$$

By replacing (61) into (58) and after some manipulations, the utility of a rich parent investing in top-up becomes

$$u\left(x - e_j, y_r^{t+1}\right) = \gamma + \rho y_0 (1 + k_1 g_B)(1 + k_2 \phi g_U), \tag{62}$$

where the term $\gamma = a\overline{x} + b(x - \overline{x}) + \frac{b}{\beta}(\omega_j - (1-\theta)(1-\lambda_r))$ is constant with respect to α, while $\phi = (1-\theta)\left(1 - \frac{b}{a}\right) < 1$.

The FOC with respect to the share of education budget allocated to higher education is

$$\frac{\partial u\left(x - e_j, y_r^{t+1}\right)}{\partial \alpha} = \rho y_0 \left(\frac{dg_B}{d\alpha} k_1 (1 + k_2 \phi g_U) + (1 + k_1 g_B) k_2 \phi \frac{dg_U}{d\alpha} \right) = 0. \tag{63}$$

By using the definitions (1) and (2), the above FOC can be rewritten as

$$\frac{k_2\phi}{\theta}\left(1+\left(1-2\alpha\right)k_1R\right)-k_1=0. \tag{64}$$

By solving (64) for α, one obtains the optimal allocation rule (42) presented in Remark 3, when the level of initial income inequality is high. Moreover, this allocation rule is an interior solution (i.e. $a_T \in (0,1)$) if the following two conditions are satisfied

$$\begin{cases} \left.\frac{\partial u(\cdot)}{\partial \alpha}\right|_{\alpha\to 0} > 0 \Rightarrow R > \frac{\theta}{k_2\phi} - \frac{1}{k_1} \\ \left.\frac{\partial u(\cdot)}{\partial \alpha}\right|_{\alpha\to 1} < 0 \Rightarrow R > \frac{1}{k_1} - \frac{\theta}{k_2\phi} \end{cases}.$$

Figure 11 illustrates these two conditions. More specifically, the dashed line represents all the configurations such that $\left.\frac{\partial u}{\partial \alpha}\right|_{\alpha\to 0} = 0$. That is, for all combinations located above such line the FOC (64) is negative for $\alpha \to 0$, therefore parents investing in top-up prefer to allocate the entire education budget to basic education, i.e. $\alpha_T = 0$. The solid line shows all configurations such that $\left.\frac{\partial u}{\partial \alpha}\right|_{\alpha\to 1} = 0$. That is, the region below such line represents all configurations such that parents investing in top-up prefer to allocate the entire education budget to higher education, i.e. $\alpha_T = 1$. Finally, the grey region, bounded by the dashed and the solid line, corresponds to all configurations such that $a_T \in (0,1)$.

Since $\phi = (1-\theta)\left(1-\frac{b}{a}\right) < 1$, the lower the level of elitism θ, the higher is the ratio $\frac{\theta}{k_2\phi}$, (i.e. the slope of the two lines decreases), therefore the regions associated with an interior solution narrows. This result is quite intuitive as from one hand, the larger is θ, the larger is the group of parents who prefer $\alpha > \alpha_T$, because their child receives higher education without top-up. On the other hand, when θ increases, the income differential between high and low educated children decreases, then the incentive to invest in private education for parents with a medium-talented child decreases.

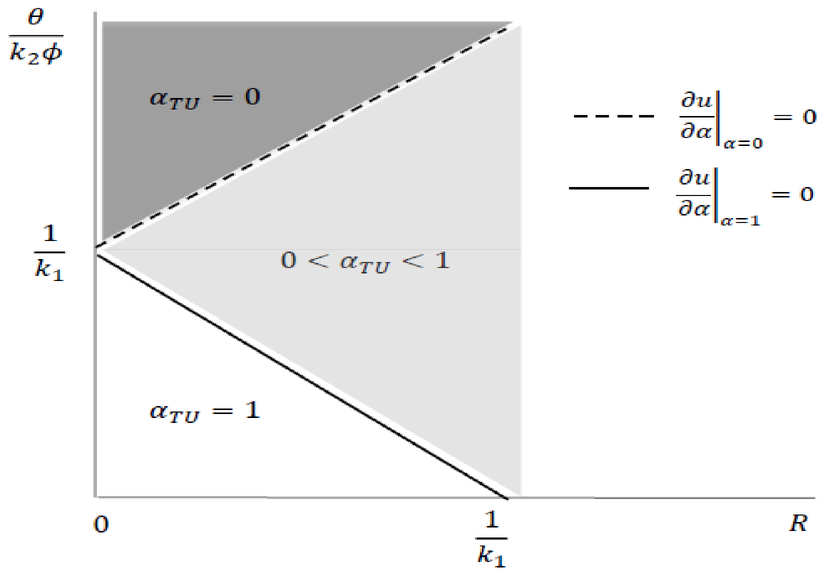

Fig 11: Allocation rule with top-up.

Medium initial income inequality. With a medium level of initial income inequality, the required admission score is

$$s^m = (1-\theta) + \beta Z\left(\frac{\theta}{b} + \frac{(1-\theta)}{a}\right) + \theta\lambda_r + (1-\theta)\,\beta\frac{(a-b)}{a}\,(x_i - \overline{x})\,, \tag{65}$$

which implies that the investment in top-up of a parents, whose child's talent is ω_j, can be written as

$$e_j^m = \frac{1}{\beta}\left((1-\theta)(1-\lambda_r) + \beta\rho k_2 g_U y_0 (1+k_1 g_B)\left(\frac{\theta}{b} + \frac{(1-\theta)}{a}\right) + (1-\theta)\,\beta\left(\frac{a-b}{a}\right)(x-\overline{x}) - \omega_j\right), \tag{66}$$

where the superscript m denotes the level of initial income inequality. By replacing (66) into (58), the utility of a parent investing in top-up is

$$u\left(x - e_j, y_r^{t+1}\right) = \gamma^m + \rho y_0 (1+k_1 g_B)(1+k_2\phi g_U)\,, \tag{67}$$

where the term $\gamma^m = a\overline{x} + b(x-\overline{x}) - (1-\theta)\,\beta\left(\frac{a-b}{a}\right)(x-\overline{x}) + \frac{b}{\beta}\left(\omega_j - (1-\theta)(1-\lambda_r)\right)$ is constant with respect to α, while $\phi = (1-\theta)\left(1-\frac{b}{a}\right) < 1$. It is interesting to note that, with a medium level of initial income inequality the investment in top-up increases with respect to the case with high inequality. However, the amount of "extra" top-up (which is measured by the term $(1-\theta)\beta\left(\frac{a-b}{a}\right)(x-\overline{x})$ into (66)) does not depend on the quality of higher education α, therefore the allocation rule maximizing the utility of a parent with a medium-talented child is given by (42), as described by Remark 3.

Low initial income inequality. When initial income inequality is low and all parents have the same marginal utility of net income, the admission score to enter higher education is

$$s^{\ell} = (1-\theta) + \beta\frac{Z}{b} + \theta\lambda_r, \tag{68}$$

while the top-up investment for the generic parent, whose child's talent is ω_j, can be written as

$$e_j = \frac{1}{\beta}\left((1-\theta)(1-\lambda_r) - \omega_j\right) + \frac{1}{b}\rho k_2 g_U y_0 (1 + k_1 g_B). \tag{69}$$

By replacing (69) into (58), one obtains that the utility of the generic parent investing in top-up, when the level of initial inequality is low, becomes

$$u\left(x - e_j, y_r^{t+1}\right) = \gamma + \rho y_0 (1 + k_1 g_B), \tag{70}$$

where the term $\gamma = a\overline{x} + b(x-\overline{x}) + \frac{b}{\beta}(\omega_j - (1-\theta)(1-\lambda_r))$ is constant with respect to α. The derivative of (70) with respect to the allocation rule is

$$\frac{\partial u\left(x - e_j, y_r^{t+1}\right)}{\partial \alpha} = \rho y_0 \frac{dg_B}{d\alpha} k_1 = \rho y_0 \alpha R k_1 < 0,$$

therefore, when the level of initial income inequality is low, the optimal allocation rule for parents investing in top-up is $\alpha_T = 0$.

Parents with a low-talented child

The utility of a rich parent with a low-talented child receiving only basic education is

$$u\left(x, y_p^{t+1}\right) = a\overline{x} + b(x-\overline{x}) + \rho y_0 (1 + k_1 g_B). \tag{71}$$

The derivative with respect to the allocation rule α is

$$\frac{\partial u\left(x, y_p^{t+1}\right)}{\partial \alpha} = \rho y_0 \frac{dg_B}{d\alpha} k_1 = \rho y_0 \alpha R k_1 < 0,$$

therefore, as presented in Remark 4, the generic parent whose child receive only basic education prefers to allocate the entire education budget to basic education, which represents a sort of insurance for their child's future income.

Appendix B: derivation of the talent threshold $\overline{\omega}$

Section 2.7.3 has introduced the threshold $\overline{\omega}_r$ denoting the talent level of the marginal child, from a rich family, for which the cost of top-up and the benefit of higher education coincide. This level is obtained by equating: *i)* the utility of the parent whose child is admitted to higher education with top-up, and *ii)* the utility of the parent whose child receives only basic education, with the entire education budget allocated to basic education. This condition, when the level of initial income inequality is high, can be formalized as

$$a\overline{x}+b\left(x-\overline{x}\right)+\frac{b}{\beta}\left(\omega_j-\left(1-\theta\right)\left(1-\lambda_r\right)\right)+\rho y_0\left(1+k_1 g_B\right)\left(1+k_2\phi g_U\right) = a\overline{x}+b\left(x-\overline{x}\right)+\rho y_0\left(1+k_1 R\right),$$

which can be rewritten as

$$\frac{b}{\beta}\left(\omega_j-\left(1-\theta\right)\left(1-\lambda_r\right)\right)+\rho y_0\left(1+k_1\left(1-\alpha\right)R\right)\left(1+k_2\phi\frac{\alpha R}{\theta}\right) = \rho y_0\left(1+k_1 R\right), \tag{72}$$

where the term on the LHS corresponds to the utility of having a high-educated child (second term) net of the top-up investment (first term), while the term on the RHS is the benefit of having a low-educated child, when the education budget is totally allocated to basic education. The threshold $\overline{\omega}_r$ is obtained by solving (72) for ω_j, that is

$$\overline{\omega}_r^h = \left(1-\theta\right)\left(1-\lambda_r\right)+\frac{\beta}{b}\left(\rho y_0 \alpha R k_1 - Z\phi\right),$$

where $Z = \rho y_0\left(1+k_1\left(1-\alpha\right)R\right)k_2\frac{\alpha R}{\theta}$, while the superscript h stands for high inequality.

With a medium level of initial income inequality, the threshold $\overline{\omega}_r$ is obtained from the following condition

$$\frac{b}{\beta}\left(\omega_j-(1-\theta)(1-\lambda_r)-(1-\theta)\beta\left(\frac{a-b}{a}\right)(x-\overline{x})\right)+\rho y_0\left(1+k_1(1-\alpha)R\right)k_2\phi\frac{\alpha R}{\theta}=\rho y_0 k_1\alpha R, \tag{73}$$

which implies that

$$\overline{\omega}_r^m=(1-\theta)(1-\lambda_r)+(1-\theta)\beta\left(\frac{a-b}{a}\right)(x-\overline{x})+\frac{\beta}{b}\left(\rho y_0\alpha R k_1-Z\phi\right).$$

While when the level of initial income inequality is low, the condition to obtain $\overline{\omega}_r$ is

$$\frac{b}{\beta}\left(\omega_j-(1-\theta)(1-\lambda_r)\right)+\rho y_0\left(1+k_1(1-\alpha)R\right)=\rho y_0\left(1+k_1 R\right), \tag{74}$$

with the threshold $\overline{\omega}$ defined as

$$\overline{\omega}_r^\ell=(1-\theta)(1-\lambda_r)+\frac{\beta}{b}\rho y_0\alpha R k_1.$$

By comparing the threshold $\overline{\omega}_r^i$, with the cutoff $\widehat{\omega}_r^i$, denoting the marginal child who can reach the admission score with the maximal top-up, one obtains that for each level of initial income inequality

$$\overline{\omega}_r^i-\widehat{\omega}_r^i=\frac{\beta}{b}\rho y_0\alpha R k_1>0.$$

This difference represents the share of parents with a medium-low talented child who prefer to allocate the entire education budget to basic education, even if their child may enter higher education with supplemental investment in private education. For these parents the cost of the top-up, expressed as a lower current income, is larger then the benefit of higher education, expressed in terms of a higher future child's income. The share of this group of parents increases when the quality of higher education improves.

Appendix C: The political economy equilibrium

This Appendix shows the conditions for the political economy equilibrium described by Proposition 2 and Proposition 3. Recall that Proposition 2 describes the political economy equilibrium when the level of initial income inequality is either high or medium. However, this Appendix reports only the computations for the case with high inequality. For the case of medium inequality, the only difference is the presence of the constant term $\beta(1-\theta)\left(\frac{a-b}{a}\right)(x_i-\overline{x})$, which increases the thresholds $\overline{\omega}_r$ and $\widehat{\omega}_r$.

High initial income inequality

The political economy equilibrium with a high level of initial income inequality is formalized by Proposition 2. More specifically, the majority voting allocation rule is $\alpha_{MV}=0$, when more than half parents prefer to allocate the entire education budget to basic education. Recall that this group includes: *i)* all parents with a low talented child, who cannot reach the required admission threshold even with the maximal top-up (i.e. child whose talent is $\omega<\widehat{\omega}_r^h$), *ii)* a share of parents with a medium-low talented child (i.e. a child with a level of talent $\omega\leq\overline{\omega}_r^h$), who can enter higher education with top-up, but the cost of this investment is not profitable.

The condition to have $\alpha_{MV}=0$, described by point *(i)* of Proposition 2, can be formalized as

$$(1-\theta)(1-\lambda_r)>\frac{\beta}{b}\rho y_0 R\alpha_T\left(\frac{k_2}{\theta}(1+k_1(1-\alpha_T)R)\phi-k_1\right)+\frac{1}{2}, \tag{75}$$

by replacing the definition α_T given by (42) and after some manipulations, one obtains that the political economy equilibrium is $\alpha_{MV}=0$ if

$$\frac{\beta}{b}\frac{\rho y_0}{4\theta}\frac{(Rk_1k_2\phi+k_2\phi-k_1\theta)^2}{k_1k_2\phi}+\frac{1}{2}<(1-\theta)(1-\lambda_r),$$

where $\phi=(1-\theta)\left(1-\frac{b}{a}\right)$.

The political economy allocation rule is the alternative α_T, (point *(ii)* of Proposition 2) when

$$\begin{cases} (1-\theta)(1-\lambda_r)\leq\frac{1}{2}+\frac{\beta}{b}\frac{\rho y_0}{4\theta}\frac{(Rk_1k_2\phi+k_2\phi-k_1\theta)^2}{k_1k_2\phi} \\ (1-\theta)(1-\lambda_r)>\frac{1}{2}-\left(\theta+(1-\theta)\frac{b}{a}\right)\frac{\beta}{b}\frac{\rho y_0}{4\theta}\left(R+\frac{1}{k_1}-\frac{\theta}{k_2\phi}\right)\left(Rk_1k_2+k_2+\frac{\theta k_1}{\phi}\right) \end{cases},$$

where the first row requires that $\overline{\omega}|_{\alpha=\alpha_T}\leq\frac{1}{2}$, while the second row implies that $\widetilde{\omega}_r|_{\alpha=\alpha_T}>\frac{1}{2}$.

If the median voter is a rich parent with a high-medium talented child, the majority voting allocation rule is $\alpha \in (\alpha_{TU}, \alpha_h)$, (i.e. point *(iii)* of Proposition 2), which requires the following condition

$$\begin{cases} (1-\theta)(1-\lambda_r) \leq \frac{1}{2} - \left(\theta + (1-\theta)\frac{b}{a}\right)\frac{\beta}{b}\frac{\rho y_0}{4\theta}\left(R + \frac{1}{k_1} - \frac{\theta}{k_2\phi}\right)\left(Rk_1k_2 + k_2 + \frac{\theta k_1}{\phi}\right) \\ (1-\theta)(1-\lambda_r) \geq \frac{1}{2} - \left(\theta + (1-\theta)\frac{b}{a}\right)\frac{\beta}{b}\frac{\rho y_0}{4\theta}\left(R + \frac{1}{k_1} - \frac{\theta}{k_2}\right)(Rk_1k_2 + k_2 + \theta k_1) \end{cases},$$

where the first row represents the case $\widetilde{\omega}_r|_{\alpha=\alpha_T} \leq \frac{1}{2}$, while the second row corresponds to the case $\widetilde{\omega}_r|_{\alpha=\alpha_H} \geq \frac{1}{2}$.

Finally, the political economy equilibrium is decided by the group of parents with a high-talented child, i.e. $\alpha_{MV} = \alpha_H$ (point *(iv)* of Proposition 2) if the following condition is satisfied

$$(1-\theta)(1-\lambda_r) < \frac{1}{2} - \left(\theta + (1-\theta)\frac{b}{a}\right)\frac{\beta}{b}\frac{\rho y_0}{4\theta}\left(R + \frac{1}{k_1} - \frac{\theta}{k_2}\right)(Rk_1k_2 + k_2 + \theta k_1).$$

Low initial income inequality

The political economy equilibrium with a low level of initial income inequality is presented by Proposition 3. Recall that, when initial inequality is low, and all parents have the same marginal utility of net income, $\alpha_T = 0$. The political economy equilibrium is $\alpha_{MV} = 0$ if more than half parents have a child receiving only basic education (point *(i)* Proposition 3). This condition requires that

$$(1-\theta)(1-\lambda_r) > \frac{1}{2}. \tag{76}$$

The entire education budget is allocated to basic education if the elitism of higher education is $\theta < \frac{1-2\lambda}{2-\lambda}$. That is, the threshold $\theta^{\ell} = \frac{1-2\lambda}{2-\lambda}$ can be interpreted as the maximum level of elitism that a society may tolerate. In other words, with a low level of income inequality, if the fraction of student admitted to higher education is lower than θ^{ℓ}, the entire education budget is allocated to basic education.

The majority voting equilibrium is a $\alpha_{MV} \in (0, \alpha_H]$ (point *(ii)* of Proposition 3) if the following condition holds

In particular, the allocation rule obtaining the preference of at least half of the voters is $0 < \alpha_{MV} < \alpha_h$ if

$$(1-\theta)(1-\lambda_r) \leq \frac{1}{2} \leq \left.\widetilde{\omega}_r^{\ell}\right|_{\alpha=\alpha_H},$$

where the term

$$\left.\widetilde{\omega}_r^{\ell}\right|_{\alpha=\alpha_H} = (1-\theta)(1-\lambda_r) + \frac{\beta}{b}\frac{\rho y_0}{4\theta}(Rk_1k_2 + k_2 + \theta k_1)\left(R + \frac{1}{k_1} - \frac{\theta}{k_2}\right)$$

Lastly, the optimal allocation rule under majority voting is $\alpha_{MV} = \alpha_H$ if more the median voter is a parents with a high talented child, that is if

$$(1-\theta)(1-\lambda_r) > \frac{1}{2} - \frac{\beta}{b}\frac{\rho y_0}{4\theta}(Rk_1k_2 + k_2 + \theta k_1)\left(R + \frac{1}{k_1} - \frac{\theta}{k_2}\right).$$

References

[1] Acemoglu, D. and Robinson, J. (2000). Why Did the West Extend the Franchise? Growth, Inequality and Democracy in Historical Perspective. Quarterly Journal of Economics 115 (4): 1167-99.

[2] Bourguignon, F. and Verdier, T. (2000). Oligarchy, democracy, inequality and growth. *Journal of Development Economics,* 62:285-313.

[3] Brossard, M. and Foko, B. (2006). Education's Impact on Economic Growth. Washington D.C. and Dakar: The World Bank and UNESCO-P$\widehat{o}$le de Dakar.

[4] Brunori, P., Ferreira, F.H.G. and Peragine, V. (2013). Inequality of Opportunity, Income Inequality and Economic Mobility: Some International Comparisons. In Eva Paus (ed.), *Getting Development Right: Structural Transformation, Inclusion, and Sustainability in the Post-Crisis Era.* Palgrave Macmillan, New York, 2013.

[5] De Fraja, G. (2004). Education and redistribution. *Rivista di Politica Economica*, V-VI: 3-44.

[6] Di Gioacchino, B. and Sabani, L.(2009). Education policy and inequality: A political economy approach. *European Journal of Political Economy*, 25:463-478.

[7] Fernandez, R. and Rogerson, R. (1995). On the political economy of education subsidies. *Review of Economic Studies*, 62:249-262.

[8] Ferreira, F.H.G. (2001). Education for the masses? The interaction between wealth, educational and political inequalities. *Economics of Transition,* 9:533-552.

[9] Gemmel, N. (1996). Evaluating the impacts of human capital stocks and accumulation on economic growth. Some new evidence. *Oxford Bullettin of Economics and Statistics*, 58: 9-28.

[10] Gradstein, M. (2003). The Political Economy of Public Spending on Education, Inequality, and Growth. World Bank Policy Research Working Paper 3162. World Bank Washington D.C.

[11] Grossman, H.I. and Kim, M. (2003). Education Policy: Egalitarian or Elitist?. *Economics & Politics*, 15:225-246.

[12] Hidalgo-Hidalgo M. and Iturbe-Ormaetxe I. (2012). Should we transfer resources from college to basic education? *Journal of Economics*, 105:1-27.

[13] Keller, K.R.I. (2010). How can education policy improve income distribution? An empirical analysis of education stages and measures on income inequality. *The Journal of Developing Areas*, 43:51-77.

[14] Naito, K. and Nishida, K. (2017). Multistage public education, voting and income distribution. *Journal of Economics*, 120:65-78.

[15] Patron, R. and Vaillant, M. (2012). Public Expenditure on Education and Skill Formation: Is There a Simple Rule to Maximize Skills? *Oxford Development Studies*, 40:261-271.

[16] Psacharopoulos, G. and Patrinos, H.A.(2004). Returns to investment in education: a further update. *Education Economics*, 12 (2): 111-134.

[17] Psacharopoulos, G. and Patrinos, H.A.(2018). Returns to Investment in Education: A Decennial Review of the Global Literature. *Education Economics*, 26: 445-458.

[18] Su, X. (2004). The allocation of public funds in a hierarchical educational system. *Journal of Economics Dynamics and Control*, 28:2485-2510.

[19] Su, X. (2006). Endogenous determination of public budget allocation across education stages. *Journal of Development Economics*, 81:438-456.

[20] UNICEF (2015). The Investment Case for Education and Equity. New York: UNICEF.

[21] Yang, L. and McCall, B. (2014). World education finance policies and higher education access: A statistical analysis of World Development Indicators for 86 countries. *International Journal of Educational Development*, 35:25-36.