Ca' Foscari University of Venice

Department of Economics

Working Paper

Marius Cristian Frunza
and
Dominique Guégan

Is the Bitcoin Rush Over?

ISSN: 1827-3580
No. 10/WP/2018

Working Papers
Department of Economics
Ca' Foscari University of Venice
No. 10/WP/2018
ISSN 1827-3580

Is the Bitcoin Rush Over?

Marius Cristian Frunza
LabEx ReFi; Schwarzthal Kapital, France

and

Dominique Guégan
University Paris 1 Pantheon Sorbonne; Ca' Foscari Unversity Venice; IPAG Business School; LabEx ReFi

Abstract
The aim of this research is to explore the econometric features of Bitcoin-USD rates. Various non-Gaussian models are fitted to daily returns in order to underline the unique characteristics of Bitcoin when compared to other more traditional currencies. Market efficiency hypothesis is tested further, and the main reasons for breaches in efficiency are discussed. The main goal of the paper is to assess the presence of bubble effects in this market with customized tests able to detect the timing of various bubbles. The results show that the Bitcoin prices had two episodes of rapid inflation in 2014 and 2017.

Keywords
Bitcoin, crypto-currencies, bubbles, market efficiency, timeseries modeling

JEL Codes
C2, C4, C5

Address for correspondence:
Dominique Guegan
Department of Economics
Paris-1 Pantheon-Sorbonne
106 bd de l'Hopital
75013 Paris, France
e-mail: dominique.guegan@univ-paris1.fr

The Working Paper Series
is available only on line
(http://www.unive.it/pag/16882/)
For editorial correspondence, please contact:
wp.dse@unive.it

Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta San Giobbe
30121 Venice Italy
Fax: ++39 041 2349210

Is the Bitcoin Rush Over?

Dominique Guegan[1,2,3,5], Marius Cristian Frunza[4,3]

Abstract

The aim of this research is to explore the econometric features of Bitcoin-USD rates. Various non-Gaussian models are fitted to daily returns in order to underline the unique characteristics of Bitcoin when compared to other more traditional currencies. Market efficiency hypothesis is tested further, and the main reasons for breaches in efficiency are discussed. The main goal of the paper is to assess the presence of bubble effects in this market with customized tests able to detect the timing of various bubbles. The results show that the Bitcoin prices had two episodes of rapid inflation in 2014 and 2017.

Keywords: Bitcoin, crypto-currencies, bubbles, market efficiency, timeseries modeling

1. Introduction

Fortune magazine published in 2014 an igniting article of Jeffrey Robinson [6], one of world's biggest financial crime authors that tackled in a harsh way cryptocurrecies and especially Bitcoin. The article made a very pessimistic forecast for Bitcoin's legacy. The French journal "Marianne" was even more direct qualifing Bitcoin as the new scam 'à-la-mode' on internet [7]. Despite a negative and reluctant reception from part of the public opinion, crypto-currencies are without any doubt the main financial innovation, since the credit derivatives. Many libertarian economics see this new 'virtual' currency

[1]Paris-1 Panthéon-Sorbonne, 106 bd de l'Hopital, 75013, Paris, France
[2]IPAG, 184 bd. Saint-Germain, 75006, Paris, France
[3]LABEX ReFi, 79 av de la Republique, 75011, Paris, France
[4]Schwarzthal Kapital,176 av Charles de Gaulle, 92200, Neuilly sur Seine, France
[5]University Ca'Foscari, Venezia, Italy

[6]Jeffrey Robinson argues that the Bitcoin movement will end in tears for the little guy. http://fortune.com/2014/10/24/bitcoin-fraud-scam/

[7]Marianne, Bitcoin, The giant scam on Internet http://www.marianne.net/Bitcoin-l-arnaque-geante-sur-internet_a231609.html

as the new Holy Grail of a 21^{st} century global economy trapped in a long recovery post-crisis scenario. Its advocates pledge for its advantages as sources of progress in the electronic economy and also from democratizing the global trade and the access to currencies.

Bitcoin, from far the most popular made surface in 2013 (Figure 1) when its exchange rate with the US dollar rallied from almost nothing to 1,000 dollars for one Bitcoin, thereby being most likely the first virtual financial bubble.

Plunging back in the history it appears that alternative payment methods are not new and many solutions like PayPal, Apple Pay and Google Wallet, which are still based on fiat currency represent viable solutions mainly for the e-commerce. Beyond these digital ways of using fiat money new digital currencies have risen over the past two decades, crypto-currencies being only a sub-category of digital currencies (Lee (2015)).

Attempts for creating a distributed digital currency date from 1990 with DigiCash Inc. founded by David Chaum (Chaum et al. (1990)). DigiCash introduced eCash as probably first cryptocurrency. Despite some initial popularity eCash did not survive to the 2000 Internet bubble (Frunza (2015)).

When in 1971 Nixon administration liberated the US dollar from the Breton Woods' covenant which implied a monetary mass backed by gold, many economist predicted the beginning of country's economic decline. Nixon's idea that dollar is backed by confidence, remained one of America's fundamental doctrine. And yet investors had appetite for a currency backed by gold and the opportunity came with the Internet era in the early 2000 when digital gold currencies made surface. Most of those second generation digital currencies like iGolder, gbullion and e-gold, were in fact electronic money backed by one ounce of gold which were stored for a fee. Their legacy was short as the companies that ran those currencies were either shut down by the Federal Government for various offenses or faded away die to heavy regulatory burdens (Frunza (2015)).

The main motivation however behind this study is to assess the sharp explosions in Bitcoin's price that occurred in 2014 and 2017. The econometric features and the price efficiency are channeling consistent facts about economic nature of Bitcoin in relation with others investments.

These aspects were explored by previous researches including MacDonell (2014) who used autoregressive moving average functions to explain trading values, then applies log-periodic power law models in an attempt to predict crashes. Bouoiyour et al. (2016) used empirical mode decomposition showed that despite the fact Bitcoin is usually labelled as a purely speculative asset, it is extremely driven by long-term fundamentals (above one year). Dyhrberg (2016) tested the hedging capabilities of Bitcoin by applying the asymmetric GARCH methodology showing that Bitcoin can clearly be used as a hedge against stocks in the Financial Times Stock Exchange Index.

Another crucial issue related to the efficiency of the Bitocoin price is the role of speculation. Blau (2017) tested whether the unusual level of Bitcoin's volatility is attributable to speculative trading. Dwyer (2015) explained how the use of these technologies and limitation of the quantity produced can create an equilibrium in which a digital currency has a positive value and also summarizes the rise of 24/7 trading on computerized markets in Bitcoin in which there are no brokers or other agents. tet

This paper enriches the literature related to Bitcoin's econometric features and explores its econometric features . When looking to its statistical features Bitcoin seems closer in nature to a commodity than to classic currency. The efficiency tests support this finding. The main results are arround the testing for the presence and the timing of bubbles in the Bitcoin/USD rates. The paper is organized as follows: *Section 2* discusses from a qualitative point of view the nature of Bitcoin, *Section 3* explains the particularities of Bitcoin prices and explores the market efficiency test, *Section 4* describes the methodology for bubble testing introduced by Phillips et al. (2013) , *Section 5* presents the results of bubbles tests, *Section 6* concludes.

2. Bitcoin: Is Bitcoin a new currency?

A first particularity of Bitcoin compared to other digital currencies relies in the fact that it uses open source software which it is not owned by a company and there is no legal entity behind it (Lee (2015)). Nevertheless a dedicated team that does the maintenance of the software does exist. The software that allows the interface with the Bitcoin universe can be downloaded freely, and the system runs through a decentralized and fully distributed peer-to-peer network.

This implies that all hardwares terminal involved are connected to each other and each terminal can leave and rejoin the network upon convenience, and will later accept the information supplied by other terminals as the authoritative record. The basic idea of this way of functioning revolves around the concept of *blocks*, that incorporate the information about the previous validated transactions. In the physical currency world this would mean that the holder of a coin or a note would be able to trace all the previous owners of the money since inception and all the transactions. The complete history of transactions is stored such as anyone can verify who is the owner of any particular group of coins. The blocks are aggregated in historical order in a blockchain. The number of transactions in a block is limited in size at 1,000,000 bytes to support quick propagation and reduced anomalies. The size of each transaction is determined by the number of inputs and outputs of that transaction. The transaction information is included in the body of a block. A Bitcoin holder when is connected to the system and reads the inputs from the blockchain has the history of what happened previously in terms of transactions. The blockchain is thus like a general ledger, carrying the track of transactions and available to everybody at any time.

Few studies including Burniske and White (2017) assessed Bitcoin's and generally crypto-currencies's economic nature. When looking at the historical time series of Bitcoin/USD exchange rate it can be easily observed that Bitcoin appreciated massively since 2013 as reflected in Figure 1. The currency exhibits jumps and regime-changes, due to a multitude of factors, like technology advances, new arrivals in the mining arena, and changes in the confidence for both crypto- and real currencies. At the dusk of the financial crisis crypto-currencies appeared as a solution in order to provide with an alternative tool to the classic failing financial system. The increasing lack of confidence in the banking system that culminated with Lehman default in September 2008 and the perspective of deposit holder to lose their economies, inflamed the speculation around crypto-currencies. They are perceived by some as a *deus-ex-machina* able to deal with the current situation. Crypto money bypasses not only the financial system but also the governmental empower related to the financial system. For these reasons Bitcoin and the similar underlyings represent more than a simple currency.

Scholars seem to find consensus (Selgin (2013)) in considering Bitcoin and generally virtual currencies as a "synthetic" commodity because it shares features with both commodity money and fiat money. Bitcoin offers to its owner an alternative that carries value in the same way stamps or art objects do. Thus Bitcoin could be more like a digital commodity that has a circumstantial intrinsic value, related to investor propensity towards it. The only difference with commodities is that it does not carry physical/real value (beside the value of the hardware used for mining). Bitcoin could be perceived as virtual good or services used for transactions as simple as the electricity or gas is used for making functioning houses and industries. This argument juxtaposed with the econometric features discussed further in Table 1, puts Bitcoin closer to a commodity than a currency.

Bitcoin offers also to its owner an unique right to exert financial activities at international level without passing through the classic system. In this new-system there are or there should be no issue related the country of residence of the parties and the regulation specific to those jurisdictions. The new Matrix is libertarian and equally in terms of rights to trade or transfer money despite embargo or sanctions that a country, corporation or group of individuals could face. At this point the *Matrix* has no *Mr. Smith* to deal with less compliant miners. This point makes Bitcoin similar to a right to transfer freely value in the same way CO_2 or SO_2 allowances give to its owners the right to pollute.

2.1. Econometric models

Statistics of the Bitcoin-USD rate (Table 1) underline that the returns exhibit strong kurtosis and high variance. Also the time series show a positive significant asymmetry. The Jarque-Bera test indicates non-Gaussian features. In terms of distribution fit the Normal Inverse Gaussian looks like the best candidate, as revealed in Figure 2.

Metric	**Value**
Mean	0.004289975
Maximum	0.6418539
Minimum	-0.4783052
Standard deviation	0.05841347
Skewness	1.485072
Kurtosis	21.60493
Jarque-Bera test	54327 (p-value =0.00)

Table 1: **Descriptive statistics for the daily returns Bitcoin-USD exchange rate**: The series exhibit strong kurtosis and high variance. The Jarque-Bera test indicates non-Gaussian features.(January 2010 -February 2018)

When looking at the annualized volatility of the Bitcoin-USD exchange rate exhibited in Figure 3 it appears that the crypto-money has more fluctuations than other forex

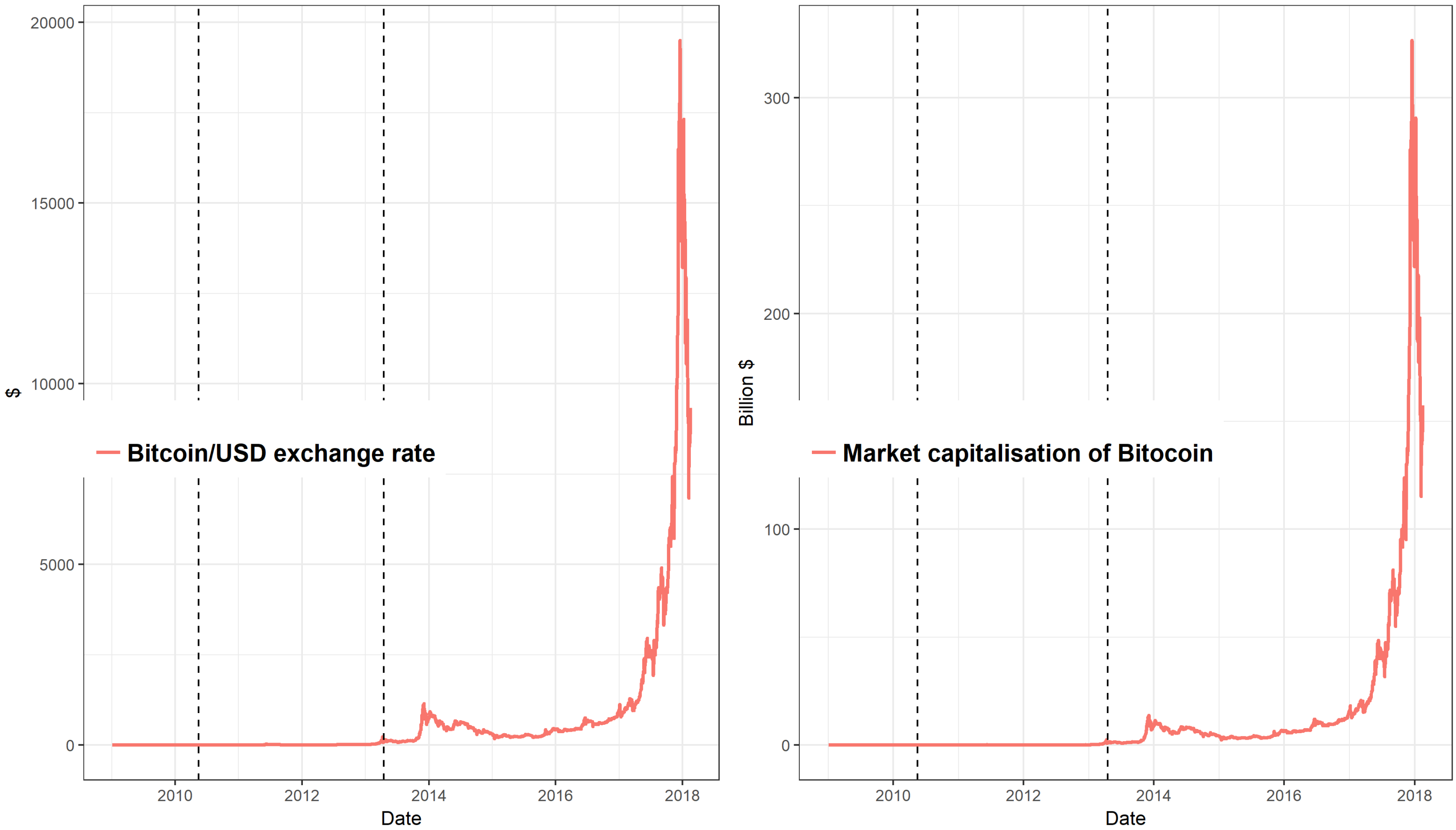

Figure 1: **Bitcoin market price evolution**: **Left** Bitcoin - USD exchange rate reached the 1,000 dollars level towards the end of 2013 and 19,000 dollars in December 2017. **Right** Bitcoin market capitalization in million USD dollars is mainly driven by the exchange rate.

markets. Even the US dollar to Russian ruble rate since the Donbass crisis has lower volatility levels (arround 50%) than Bitcoin-USD. European CO2 pollution rights and British power prices have volatility levels comparable to Bitcoin/USD rate, reinforcing the above mentioned hypothesis about Bitcoin's nature.

The analysis of the persistence in returns with the auto-correlation function reveals a strong effect of volatility clustering (Figure 4), similar to what can be observed in many commodities markets from the energy complex.

Table 2 shows the results of fitting GARCH models with various innovations to the daily returns of Bitcoin-USD rate. The best fit in terms of Bayesian Information criteria(BIC) corresponds to the GARCH model with NIG innovations, thereby underlining the fat-tails effects.

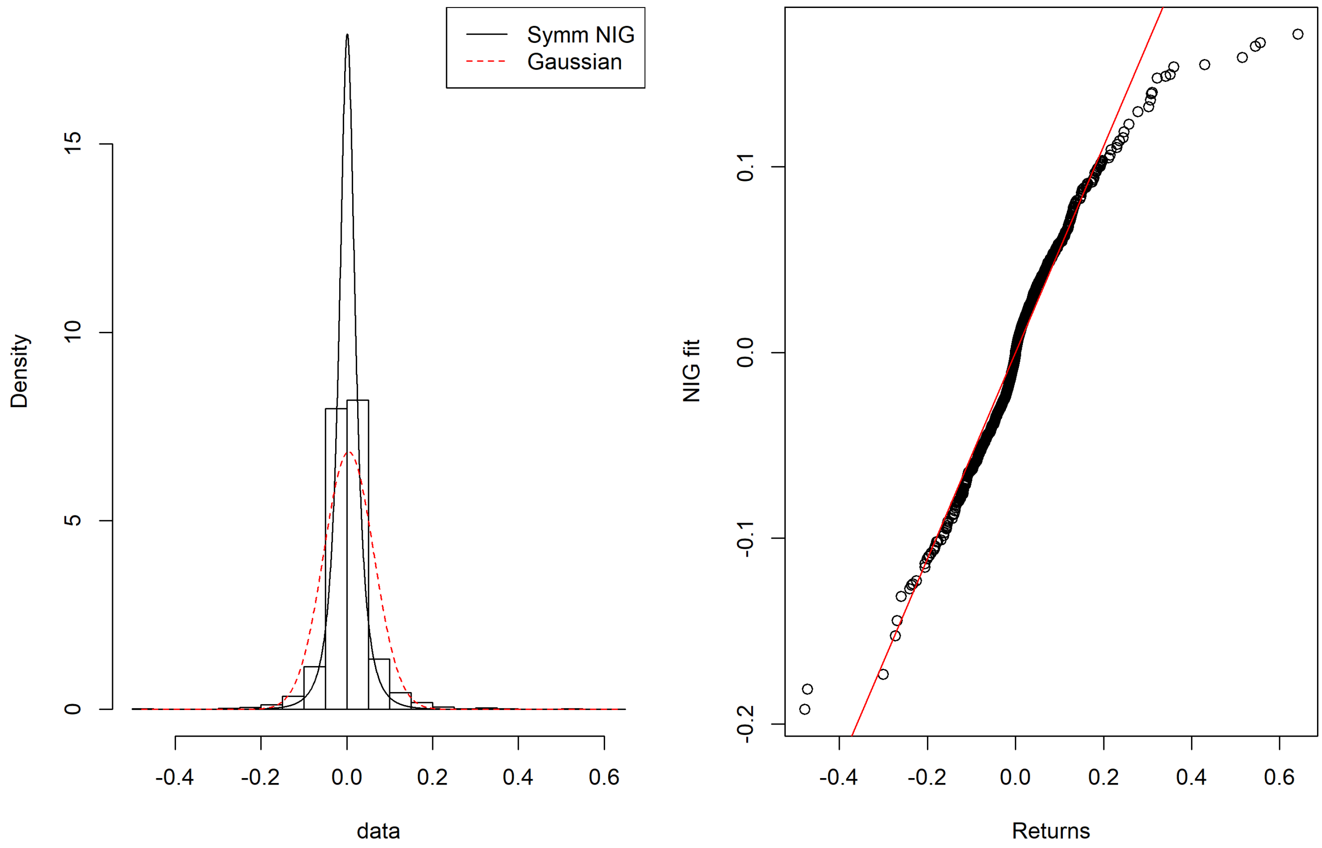

Figure 2: **Distribution fitting for the Bitcoins - USD daily returns. Left**: Histogram of the fit for the Normal Inverse Gaussian compared to the Gaussian distribution. **Right**: QQ plot for the observed returns against the NIG fit

A deposit holder in a specific currency, bears in mind many of the aspects related to this very basic investment. Fist the perspective of the currency and of the underlying economy, second the interest rate and last but not the least the creditworthiness of the banks taking the deposit if the bank is located in currency domestic country. The strengthening of the American dollar during 2014 compared to the European currency is a very good example to underline the first point. Since the Eurozone crisis American economy observed a faster and stronger recovery relative to the European Union and many analysts forecast the US dollar going towards parity with the Euro. Thus based on this appreciation one could expect to see deposit flight towards the US currency.

In the case of a crypto-currency it would be very difficult or almost impossible to make any judgments regarding the economy that backs the currency. In fact the

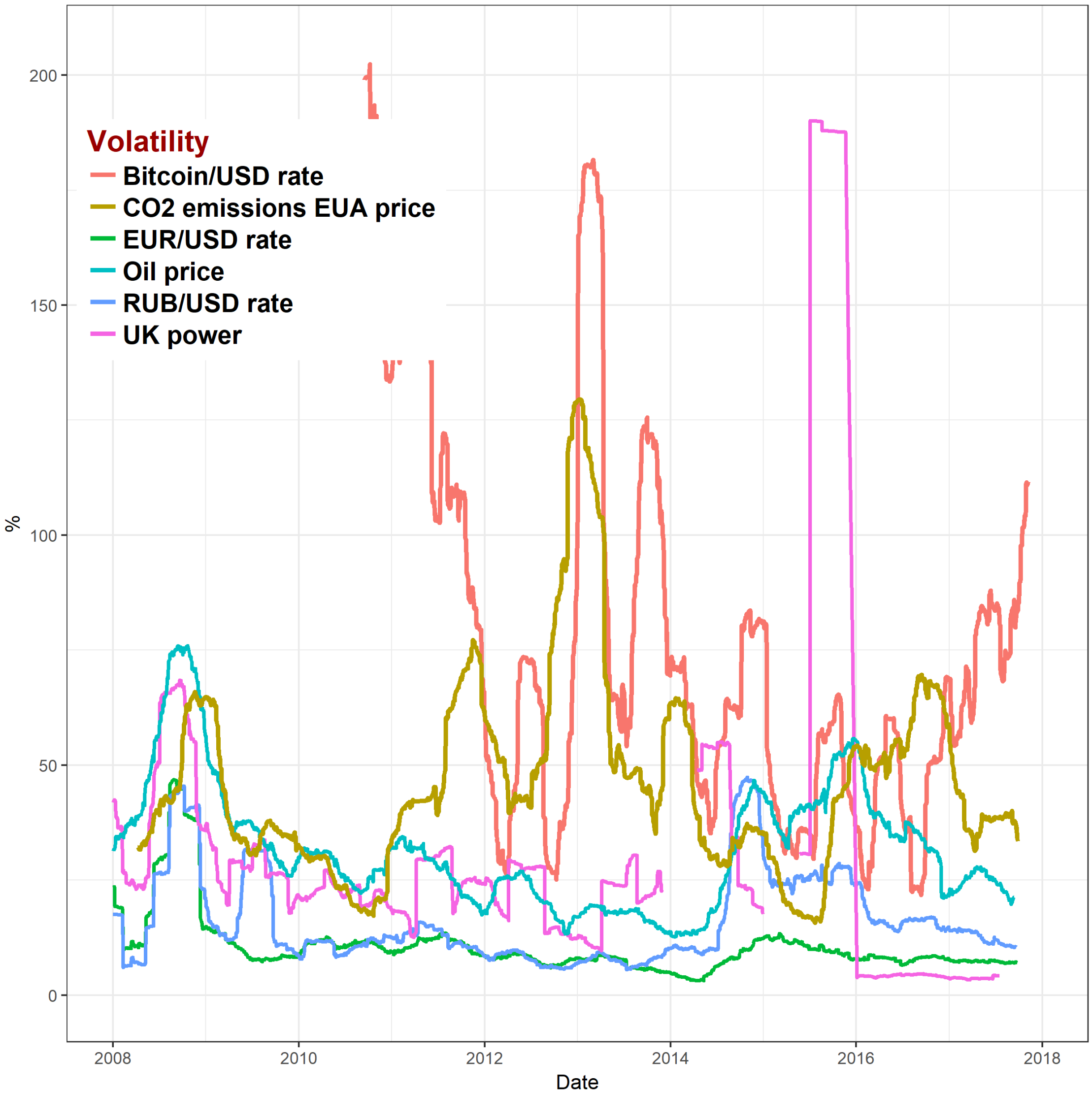

Figure 3: Volatility of the Bitcoin USD exchange rate : Benchmark with other underlyings

only reasoning would be linked to the degree of confidence merchant have towards that particular currency. Interest rates are another argument for holding deposit in fiat currency, but in the case of crytpo-currencies, interest rate seems to be very complex topic. Currently a Bitcoin account holder does not receive interest in the same way a Yen deposit holder does.

The point of the deposit guarantee scheme which is proposed for almost all developed countries for deposits lower than 100 thousands dollars(euros) is out of scope in the case of the crypto-currencies, at least until banks will adopt one of them.

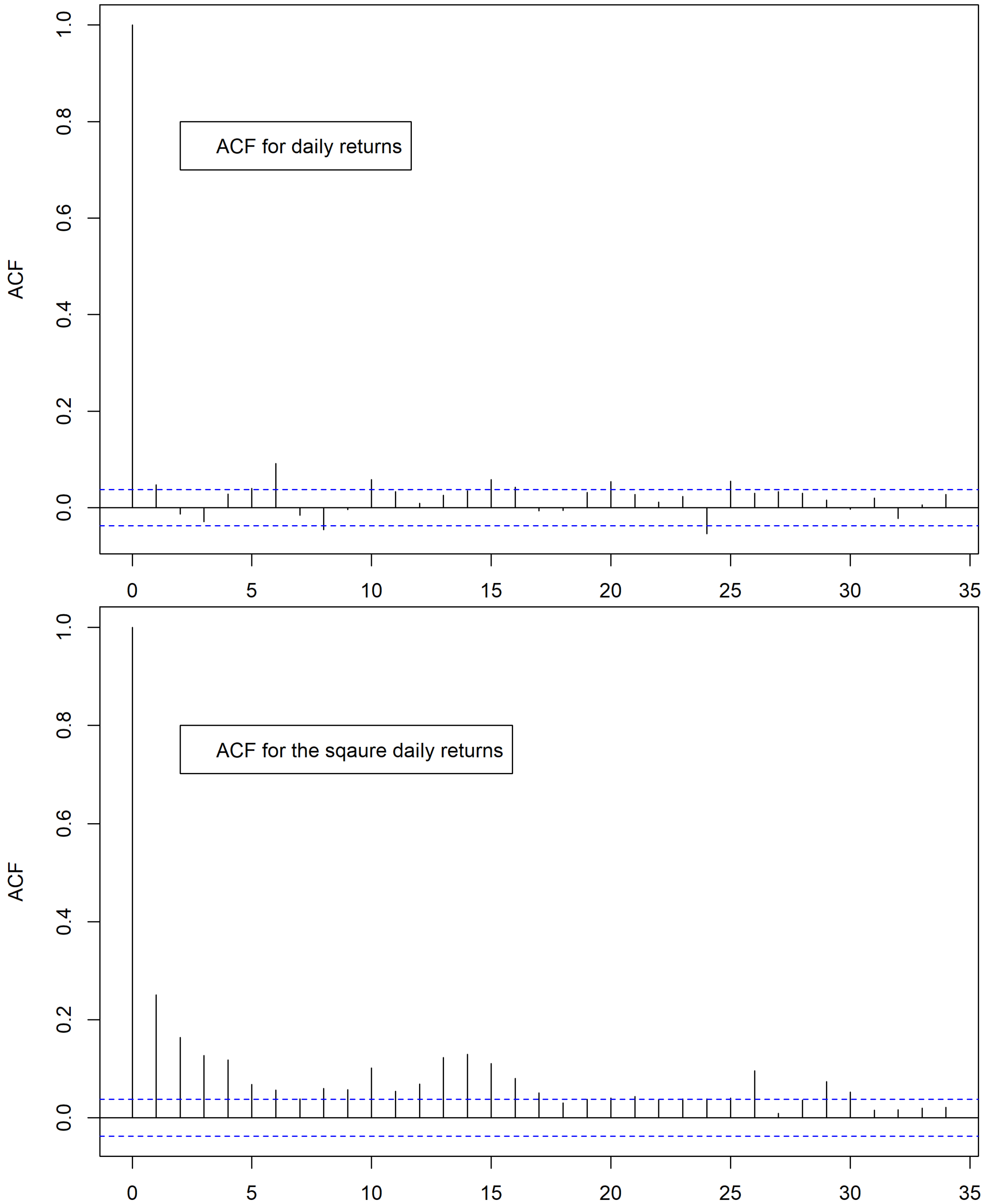

Figure 4: **Autocorrelation in the Bitcoin USD rates**. **Above:** ACF for the daily returns. **Below:** ACF for the squared daily returns

So the question relative to the true nature of Bitcoin (or other virtual/crypto-currencies) is crucial before entering further discussions about . Looking to its pure econometric feature it can be observed that Bitcoin is as far away from the features of

Parameter	GARCH			GARCH-NIG			GARCH-STD		
	Value	**Std**	**p-value**	**Value**	**Std**	**p-value**	**Value**	**Std**	**p-value**
ω	0.000068	0.000037	0.066	0.000052	0.000023	0.025	0.000047	0.000022	0.03
α	0.158890	0.027404	0.00	0.196502	0.023891	0.00	0.198569	0.022786	0.00
β	0.838407	0.027757	0.00	0.802498	0.032198	0.00	0.800431	0.032863	0.00
ν							3.111196	0.119759	0.00
ζ				0.185832	0.033286	0.00			
ξ				0.324860	0.040720	0.00			
LL	4688.52			**5241.528**			5195.99		
BIC	-3.4186			**-3.8171**			-3.7867		

Table 2: **GARCH fit parameters**: Successful fitting of GARCH model to the daily returns of Bitcoin-USD rate underlines the existence of clustering in volatility. GARCH(1,1) model with Normal Inverse Gaussian innovation provides with the best results

a classic currency as an underlying can be.

The presence of persistence in returns, clustering in volatility and fat tails in Bitcoin /USD exchange rate underline the fact that Bitcoin should be regarded differently from others currencies. From its features, Bitcoin has a lot in common with commodities, with sudden dis-ruptures in the supply demand equilibrium. A first similarity would be with commodities from the energy complex (electricity, gas , emissions). Jumps and spikes in energy (electricity) prices are explained by the fact small increases in demand can inflate the price rapidly and vice-versa oversupplies can push prices very low if there is no need for that commodity. As reveled in Figure 1 the Bitcoin/USD rate exploded in 2013 and in 2017. This would be very uncommon for a real currency even in a fast growing emerging economy. The shock observed over 2014 and 2018 when Bitcoin lost almost 50% of its value to the US dollar would have catastrophic consequences if it would happen on a real currency. Nevertheless in the commodities world this kind of variations are very frequent as regime changes in price equilibrium operate often.

2.2. Bitcoin and market efficiency

As the Bicoin becomes more popular and the flows grow in volume and frequency, the question of the market efficiency comes naturally. EUR/USD exchange is one of the most liquid markets across the globe and given its features it could be a good candidate for market efficiency. In the case of Bitcoin the dialectic is different from what one can experience in the classic markets. A Bitcoin deposit owner is generally a Bitcoin generator if he is also involved in the mining process. The mining process induces a lot of particularities that impact the market efficiency. In a classic efficient market all investor have homogeneous access to information and ability to buy and sell a fraction of the available stock. If classic currency faces high and sudden depreciation the central bank can try to address the issues by buying back currencies or altering the interest rates. Obviously in the case of the crypto-currency these aspects are not applicable. The concept of mining that currencies creates an asymmetry amongst "investors" due to the fact that not all miners have access to the same mining tools, in terms of computation speed. Thus some more advantages than others given the feature of their gear. Obviously those with stronger mining tools have a comparative advantage in the price discovery. Also each technological jump creates new sources of asymmetry amongst miners. In theory this heterogeneity due to technological aspects should be attenuated with times, when the total Bitcoin monetary mass will became stable, due to the fact that the mining will became more and more cost intense.

If technology does represent a first source of behavioral asymmetry , another source of inefficiency is the breakdown of memory and computational capacity amongst miners. From this point of view miner profiles vary strongly from a solo miner to pool mining and farms. A solo miner might use some classic technology like a central processing unit, a graphical unit or application specific Integrated Circuit (ASIC) for generating Bitcoins on a standalone basis. In theory the average time of a solo miner using a standalone computer to solve a block is around 2,000 years. Thus the only economic feasible solutions are either a massive inflation of the mining capacity or joining mining pools. Mining farms start to became a trend in countries where the cost of electricity cheap and space rent are cheap, energy consumption being the main variable cost in the mining process.

Certainly in a mining farm or mining pool would have a net informational advantage compared to solo miners or smaller pools. Higher capacity of solving the cryptographic game implies a higher rate of block solving thereby giving a better view upon the Bitcoin inflows. If a Bitcoin pool trades forex against a real currency, they will have more information about the volume of Bitcoin that would come on the market. Structurally they are better and more informed than a solo miner. This is a crucial source of market inefficiency, as a pool can generate bearish or bullish momentum on the market depending on the circumstances. Power market has similar issues in countries where there are big producers or quasi-monopolies. For example in Germany the main energy producer RWE has obviously more information on the electricity market than a small broker, due to its position of main supplier of underlying and trader.

Test name	**Statistic**	**Critical value (95%)**
Portmanteau	0.8522142	3.8
Chow and Denning	2.115319	1.959964
Wright (R1 statistics)	5.266816	1.917136
Wright (R2 statistics)	4.338164	1.903357
Wright (S1 statistics)	4.569499	1.930746
Lo and Mackinley	2.115319	1.95
Wald	4.474573	3.84

Table 3: **Tests for assessing the efficient market hypothesis (weak-form)**. The tests are performed on the time series of daily returns of Bitcoin USD exchange rate from 22/07/2010 to 21/2/2018 assuming holding period of 10 trading days

Table 3 exhibits a series of test for market efficiency (weak-form). All test statistics computed for a holding period of 10 trading days are higher than the 95% confidence level value, thereby rejecting the efficiency of the Bitcoin/USD rate from the following perspectives .

We recall the Lo and MacKinlay (1988) test for the random walk process, where they used stock-market returns. This involves the use of specification tests based on variance estimates. In particular, the method exploits the fact that the variance of the increments in a random walk is linear in the sampling interval, hypothesis rejected by in the case of Bitcoin/USD rate Chow and Denning (1993) test, a generalization of the Lo and Mackinley test, obtained from the maximum absolute value of the individual statistics confirms the results. Wright (2000)'s alternative nonparametric test using signs and ranks is complementary for Lo's test. Both signs (R1 and R2) and ranks (S1) statistics reject the hypothesis of random walk. Richardson and Smith (1991) version of the Wald test and the portmanteau test Escanciano and Lobato (2009) for autocorrelation confirms also these findings. Markets do not become efficient automatically from their origin.

It is the actions of investors and various traders, sensing arbitrage opportunities and putting into effect schemes to take profit from the market, that make markets efficient.

From another point of view, bringing efficiency in the Bitcoin system is related to the mining capacity. Not all miners dispose of the same mining capacity and the mining capacity needs to increase much faster compared to the Bitcoin transactions. A double edge effect can occur. On one hand there could be a massive increase in the number of new investors that purchase and trade Bitcoin , without mining. On the other hand the mining capacity remains constant or progresses at much lower level. This could be the reason why the Bitcoin /USD became massively inefficient during 2013, when a massive inflow of demand was followed in an asymmetric manner in terms of mining ability. Figure 5 exhibits the random walk test applied over of rolling window of 200 days for a holding period of 10 trading days. The non parametric Wright test underlines the non-random walk effect that occurred in 2013.

3. Testing for bubbles

Since the South Sea Company frenzy in the early eighteen century, financial markets faced many bubbles and as many crashes, the Black Tuesday from 1929 being one of the most dramatic ones.

The features of financial bubbles are explored the academic literature. Zhao (2014) studied the unusual and puzzling stock price performance of USEC Inc., a company specialized in producing enriched uranium for nuclear plants. In July 2013 the stock price surged as much as ten times during merely sixteen trading days without apparent value-changing information being released and the hypothesis of market manipulation and speculative bubbles are analyzed.

GENG and LU (2014) studied bubble-creating stock attacks, an interesting form of market fraud which is a mixture of manipulation and speculative bubble in which speculators implicitly coordinate to pump up the stock price without any significant fundamental news and exploit behavioral-biased investors. The research provided empirical evidence in the Chinese stock market underlining that stocks with low mutual fund ownership and stocks with high average purchase costs of existing shareholders are more likely to be attacked.

Johansen et al. (1999) presented a synthesis of all the available empirical evidence

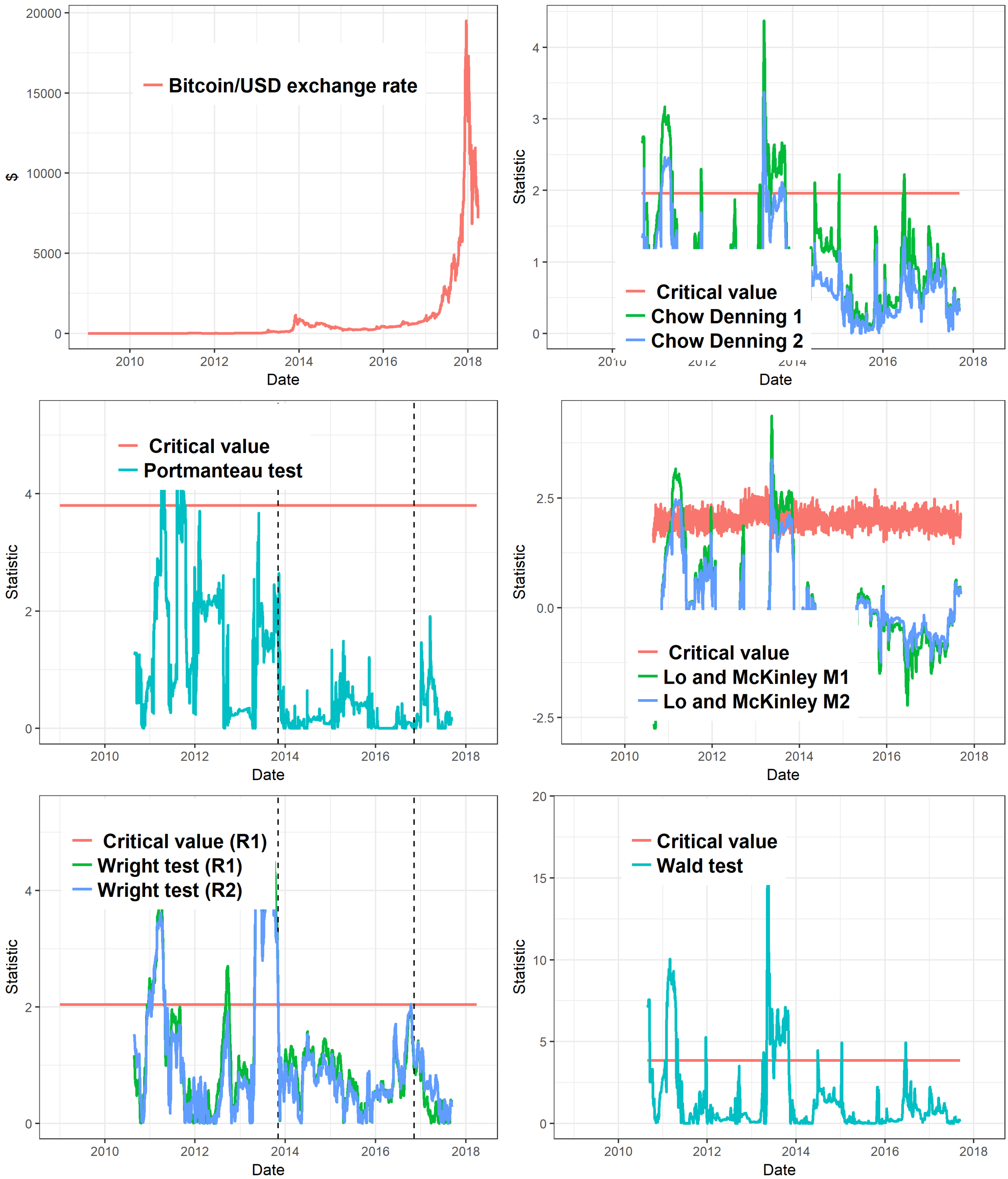

Figure 5: **Efficiency tests** applied to Bitcoin/USD daily returns for a 200 days rolling window: Portmanteau test, Wright test, Wald test, Lo and MacKinley test and Chow-Denning test

in the light of recent theoretical developments for the existence of characteristic log-periodic signatures of growing bubbles in a variety of markets

Few straightforward methods for testing a market for bubble are proposed by the recent works of Peter Phillips (Phillips et al. (2013) and Phillips et al. (2011)). These approaches come with enhanced versions of the Augmented Dickey-Fuller (ADF) test (Dickey and Fuller (1979) and Said and Dickey (1984)): Sup ADF test and Generalized Sup-ADF test.

The testing procedure for the Augmented Dickey-Fuller test for a unit root in timeseries is based on the model:

$$y_t = \alpha + \beta y_{t-1} + \gamma_1 \Delta y_{t-1} + \cdots + \gamma_p \Delta y_{t-p} + \varepsilon_t, \tag{1}$$

where p is the lag order and $\varepsilon_t \propto N(0, \sigma_t)$.

Phillips et al. (2013) improved the basic version of the ADF test with recursive approach that involving a rolling window ADF style regression implementation. If the rolling window regression sample starts from the r_1^{th} fraction of the total sample and ends at the r_2^{th} fraction of the sample, where $r_2 = r_1 + r_w$ and r_w is the fractional window size of the regression. The empirical regression model can then be written as :

$$y_t = \alpha_{r_1,r_2} + \beta_{r_1,r_2} y_{t-1} + \gamma^1_{r_1,r_2} \Delta y_{t-1} + \cdots + \gamma^p_{r_1,r_2} \Delta y_{t-p} + \varepsilon_t, \tag{2}$$

where α_{r_1,r_2} is the intercept, β_{r_1,r_2} the coefficient on a time trend and p the lag order of the autoregressive process computed on the window r_1T, r_2T. under this circumstances the unit root null hypothesis is $\mathbf{H}_0 : \beta = 1$ and the explosive root right-tailed alternative hypothesis is $\mathbf{H}_a : \beta > 1$. The ADF statistic (based on this regression is denoted by $ADF_{r_1}^{r_2}$ Phillips et al. (2011) where ADF_0^1 is the ADF statistics for the full sample. Right sided unit root tests are informative about explosive or submartingale behavior[8] in the timeseries and can be used of speculative bubble detection.

The Sup-ADF test introduced in (Phillips et al. (2011)) for single bubble detection is searching for the maximum value of the test, for all forward looking the windows on given sample. The window size r_w varies from the smallest sample window noted r_0 to

[8] It should be recall that a discrete-time submartingale is a price time series $y_1, y_2, y_3, \ldots$ is satisfying $\mathbf{E}[y_{n+1}|\Phi_n] \geq y_n$, Φ_n being the filtration with all information at moment when the price is y_n

1. In term of the formalism in equation (2) the starting point r_1 is 0; and the end point so r_2 is chosen such as the statistic $ADF_0^{r_2}$ is maximized, fact that can be written as

$$SADF(r_0) = \underbrace{sup}_{r_2 \in [r_0, 1]} ADF_0^{r_2} \tag{3}$$

A step further improvement of the Sup-ADF test is the Generalize Sup ADF (GSADF) leveraging the idea of repeatedly running the ADF test regression on sub-samples of the data in a recursive fashion. Thus, in addition to varying the end point of the regression r_2 from r_0 to 1, the GSADF test allows the starting point r_1 in to change 0 to $r_2 - r_0$. The GSADF statistic searches for the biggest ADF statistic over all possible starting point sand possible window length.

$$GSADF(r_0) = \underbrace{sup}_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0} ADF_{r_1}^{r_2} \tag{4}$$

The bottom line of this test is to search for period where the prices exhibit consistently exponentially increasing trajectories.

4. The Bitcoin rush

Gold miners were attracted through history to regions rich in gold and silver. From Dacia in antiquity, to California and Alsaka in the early years of the industrial revolution to Sierra Leone and Kirghistan in the present days, the perspective of generating quick profit from mining ignite the spirit of many generations. Similarly the foreseeable gains the crypto-currency world seem to generate a new Bitcoin rush, translated not only in high number of new comers in the crypto- world, but also a bubble of Bitcoin's value. As of 2017, the number of Bitcoin wallets users is around 10 millions compared to only 80 thousands in early 2013.

Test name	**Statistic**	**Critical value (95%)**
Sup ADF test	27.56	0.99
Generalized Sup ADF test	27.56	1.92

Table 4: **Testing for bubbles**:Sup augmented Dickey-Fuller and Generalized sup augmented Dickey-Fuller have both statistics above the 95 % critical value there by rejecting the null hypothesis of a no bubble episode in the considered Bitcoin USD times series

The assessment of the formation and propagation of bubble in markets can be exert through many ways. The results obtained from applying this bubble detection approach to the Bitcoin-US dollar rate daily returns is exhibited in Table 4. The critical values fro a 95 % confidence level are the asymptotic values communicated in Phillips et al. (2013).

Figure 6 shows the evolution over time for the statistics of two tests and indicates the corresponding timing of the bubbles. The Sup augmented Dickey-Fuller test indicates two bubbles during 2013 and one in 2017. The Generalized sup augmented Dickey-Fuller test indicates also another mini bubble during 2011.

Bubbles are period when markets change dramatically their features and also give a positive and yest biased signal to behavioral investors. During bubble periods market prices are far away from fundamentals and investors may take irrational decisions. Bitcoin is no exception.

Bubble timeline	**Initial Price**	**Peak Price**
2011-02-01 / 2011-02-18	USD 0.95	USD 1.1
2011-04-23 / 2011-06-13	USD 1.7	USD 35.00
2013-03-05 / 2013-03-05	USD 40.04	USD 237.00
2013-11-06 / 2014-01-11	USD 258.23	USD 1,151.00
2017-05-07/ 2018-01-21	USD 1,560.41	USD 19,498.68

Table 5: **Timeline of Bitcoin bubbles**: The main bubble episodes are in November 2013 - January 2014 and May 2017 -January 2018

The episodes of Bitcoin bubble resulting from the Phillips test are exposed in Table 5. The main bubble episodes are in November 2013 - January 2014, when Bitcoin peaked at USD 1,151 and May 2017 - January 2018 when Bitcoin peaked at USD 19,498.68. The 2013 bubble ended after the Mgtox event (Frunza (2015)) that led at the theft of 10% of all available Bitcoin at the time. The 2017 bubble ended when Bitcin futures where listed on the Chicago Mercantile Exchange.

The same tests applied to Ethereum prices (Figure 7) showed no bubble effect as presented in Table 6. Thus, the price inflation in Bitcoin was not reflected with the same amplitude on Ethereum markets. An extended study on the presence of bubbles on altercoins will be presented in a future paper.

Test name	Statistic	Critical value (95%)
Sup ADF test	-0.78	0.99
Generalized Sup ADF test	-0.15	1.92

Table 6: **Testing for bubbles**:Sup augmented Dickey-Fuller and Generalized sup augmented Dickey-Fuller have both statistics above the 95 % critical value there by rejecting the null hypothesis of a no bubble episode in the considered Ethereum USD times series

5. Conclusions

This paper explores the occurrence and the timing of bubbles in the Bitcoin/USD rates. Being a very new and innovative currency Bitcoin exhibits unique features, that makes it different from other currencies. The problem is studied in two steps: first the econometric features and the efficiency hypotesis are assessed and second a bubble test procedure is developed and tested on Bitcoin prices.

The results from the first part indicate that Bitcoin's econometric feature include volatility clustering and heavy tails. The weak efficiency hypothesis is breached in few occasion, during the 2013 price inflation.

The second part shows that bubble episodes occurred in 2013 and in 2017. The bubble effect on other altcoins will be described in a future paper.

Bibliography

Blau, B. M., 2017. Price dynamics and speculative trading in bitcoin. Research in International Business and Finance 41, 493–499.

Bouoiyour, J., Selmi, R., Tiwari, A. K., Olayeni, O. R., et al., 2016. What drives bitcoin price. Economics Bulletin 36 (2), 843–850.

Burniske, C., White, A., 2017. Bitcoin: Ringing the bell for a new asset class. Ark Invest (January 2017) https://research. ark-invest. com/hubfs/1_Download_Files_ARK-Invest/White_Papers/Bitcoin-Ringing-The-Bell-For-A-New-Asset-Class. pdf.

Chaum, D., Fiat, A., Naor, M., 1990. Untraceable electronic cash. In: Proceedings on Advances in cryptology. Springer-Verlag New York, Inc., pp. 319–327.

Chow, K. V., Denning, K. C., 1993. A simple multiple variance ratio test. Journal of Econometrics 58 (3), 385–401.

Dickey, D. A., Fuller, W. A., 1979. Distribution of the estimators for autoregressive time series with a unit root. Journal of the American statistical association 74 (366a), 427–431.

Dwyer, G. P., 2015. The economics of bitcoin and similar private digital currencies. Journal of Financial Stability 17, 81–91.

Dyhrberg, A. H., 2016. Hedging capabilities of bitcoin. is it the virtual gold? Finance Research Letters 16, 139–144.

Escanciano, J. C., Lobato, I. N., 2009. An automatic portmanteau test for serial correlation. Journal of Econometrics 151 (2), 140–149.

Frunza, M.-C., 2015. Solving Modern Crime in Financial Markets: Analytics and Case Studies. Academic Press.

GENG, Z., LU, X., 2014. Implicitly-coordinated stock price attack and exploitation of retail investors' behavioral biases: Widespread evidence in the chinese stock market.

Johansen, A., Sornette, D., Ledoit, O., 1999. Predicting financial crashes using discrete scale invariance. arXiv preprint cond-mat/9903321.

Lee, D., 2015. Handbook of Digital Currency: Bitcoin, Innovation, Financial Instruments, and Big Data. Academic Press.

Lo, A. W., MacKinlay, A. C., 1988. Stock market prices do not follow random walks: Evidence from a simple specification test. Review of financial studies 1 (1), 41–66.

MacDonell, A., 2014. Popping the bitcoin bubble: An application of log-periodic power law modeling to digital currency. University of Notre Dame working paper.

Phillips, P. C., Shi, S.-P., Yu, J., 2013. Testing for multiple bubbles: Historical episodes of exuberance and collapse in the s&p 500.

Phillips, P. C., Wu, Y., Yu, J., 2011. Explosive behavior in the 1990s nasdaq: When did exuberance escalate asset values? International economic review 52 (1), 201–226.

Richardson, M., Smith, T., 1991. Tests of financial models in the presence of overlapping observations. Review of Financial Studies 4 (2), 227–254.

Said, S. E., Dickey, D. A., 1984. Testing for unit roots in autoregressive-moving average models of unknown order. Biometrika 71 (3), 599–607.

Selgin, G., 2013. Synthetic commodity money. University of Georgia Economics.

Wright, J. H., 2000. Alternative variance-ratio tests using ranks and signs. Journal of Business & Economic Statistics 18 (1), 1–9.

Zhao, X., 2014. Trade-based manipulation or speculative bubble: A case study. International Business & Economics Research Journal (IBER) 13 (4), 841–852.

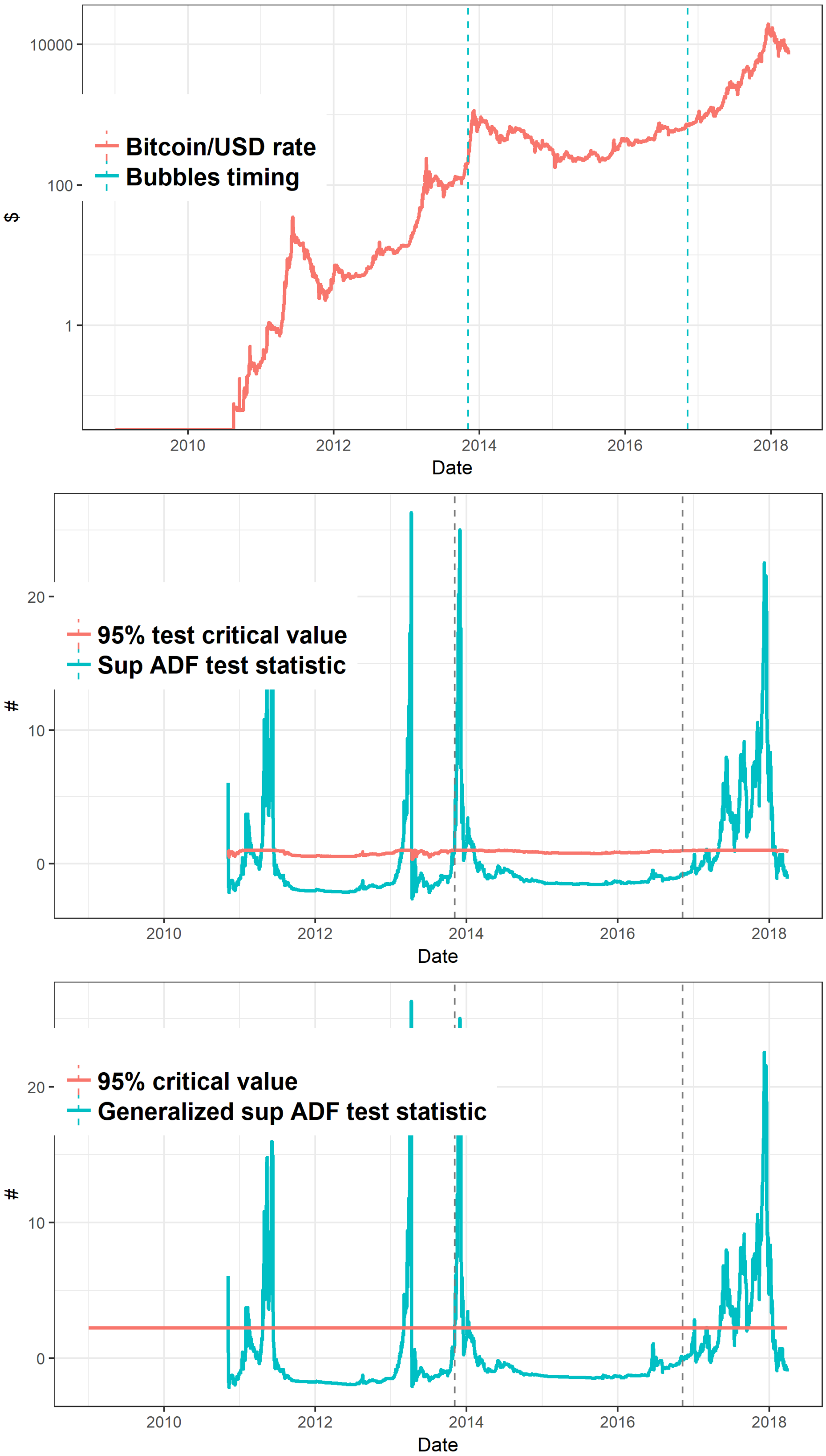

Figure 6: **Bubble detection tests on Bitcoin prices: The Sup augmented Dickey-Fuller** test indicates two bubbles during 2013. **The Generalized sup augmented Dickey-Fuller** test indicates also another mini bubble during 2012.

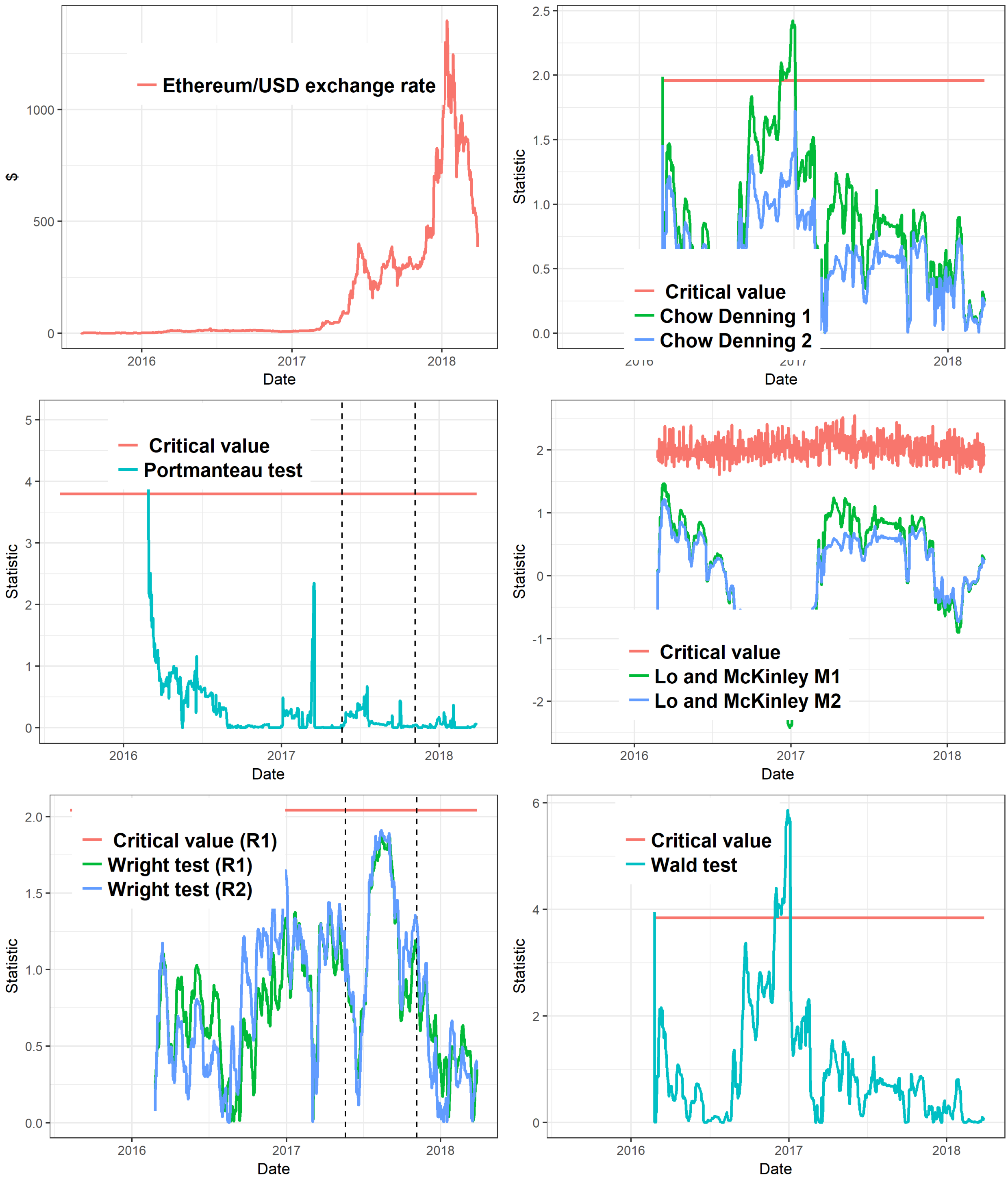

Figure 7: **Efficiency tests** applied to Ethereum/USD daily returns for a 200 days rolling window: Portmanteau test, Wright test, Wald test, Lo and MacKinley test and Chow-Denning test